Ann. Univ. Paedagog. Crac. Stud. Math. 19 (2020), 95-103
DOI: 10.2478/aupcsm-2020-0008

FOLIA 340

Annales Universitatis Paedagogicae Cracoviensis
Studia Mathematica XIX (2020)

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A new result on the quasi power increasing sequences

Abstract. This paper presents a theorem dealing with absolute matrix summability of infinite series. This theorem has been proved taking quasi β-power increasing sequence instead of almost increasing sequence.

1. Introduction

The following notations and notions will be used in this paper. If $g > 0$, then $f = O(g)$ means that $|f| < K.g$, for some constant $K > 0$ (see [5]). Let (u_n) be a sequence. We write that $\Delta u_n = u_n - u_{n+1}$, $\Delta^0 u_n = u_n$ and $\Delta^k u_n = \Delta\Delta^{k-1} u_n$ for $k = 1, 2, \ldots$ (see [5]).

Abel's transformation ([7]): Let (a_k) and (b_k) be complex sequences, and write $s_n = a_1 + a_2 + \ldots + a_n$. Then

$$\sum_{k=1}^{n} a_k b_k = \sum_{k=1}^{n-1} s_k \Delta b_k + s_n b_n.$$

Hölder's inequality ([7]): If $p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$ and $a_1, a_2, a_3, \ldots, a_n \geq 0$, and $b_1, b_2, b_3, \ldots, b_n \geq 0$, then

$$\sum_{k=1}^{n} a_k b_k \leq \left(\sum_{k=1}^{n} a_k^p\right)^{1/p} \left(\sum_{k=1}^{n} b_k^q\right)^{1/q}.$$

AMS (2010) Subject Classification: 26D15, 40D15, 40F05, 40G99.
Keywords and phrases: almost increasing sequence, Hölder inequality, infinite series, Minkowski inequality, Riesz mean, quasi power increasing sequence, summability factor.
ISSN: 2081-545X, e-ISSN: 2300-133X.

A positive sequence (d_n) is said to be almost increasing if there exist a positive increasing sequence (c_n) and two positive constants K and M such that $Kc_n \leq d_n \leq Mc_n$ (see [1]). Let $\sum a_n$ be an infinite series with its partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad \text{as} \quad n \to \infty, \qquad (P_{-i} = p_{-i} = 0,\ i \geq 1).$$

Let $A = (a_{nv})$ be a normal matrix, i.e. a lower triangular matrix of nonzero diagonal entries. Here, A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \qquad n = 0, 1, \ldots .$$

Let (φ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be summable $\varphi - |A, p_n|_k$, $k \geq 1$, if (see [14]),

$$\sum_{n=1}^{\infty} \varphi_n^{k-1} |A_n(s) - A_{n-1}(s)|^k < \infty.$$

If we take $\varphi_n = \frac{P_n}{p_n}$, then we get $|A, p_n|_k$ summability (see [21]). If we take $\varphi_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$, then we get $|\bar{N}, p_n|_k$ summability (see [2]). Also, if we take $\varphi_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n, then $\varphi - |A, p_n|_k$ summability reduces to $|C, 1|_k$ summability (see [4]).

2. Known result

Mazhar [9] has proved the following theorem.

Theorem 2.1
If (X_n) is an almost increasing sequence and the conditions:

$$|\lambda_m| X_m = O(1) \quad as \quad m \to \infty, \tag{1}$$

$$\sum_{n=1}^{m} n X_n |\Delta^2 \lambda_n| = O(1) \quad as \quad m \to \infty, \tag{2}$$

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m) \quad as \quad m \to \infty,$$

$$\sum_{n=1}^{m} \frac{|t_n|^k}{n} = O(X_m) \quad as \quad m \to \infty,$$

and

$$\sum_{n=1}^{m} \frac{p_n}{P_n} |t_n|^k = O(X_m) \quad as \quad m \to \infty$$

are satisfied, where (t_n) is the n-th $(C, 1)$ mean of the sequence (na_n), then the series $\sum a_n \lambda_n$ is summable $|\bar{N}, p_n|_k$, $k \geq 1$.

Before introducing main result, we need some notations. Let $A = (a_{nv})$ be a normal matrix. Two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ are defined as follows

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \qquad n, v = 0, 1, \ldots, \tag{3}$$

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \qquad n = 1, 2, \ldots, \tag{4}$$

and

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v, \tag{5}$$

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{6}$$

3. Main Result

There are some papers on absolute matrix summability (see [10, 11, 12, 13, 15, 16, 17, 18, 19, 20]). The aim of this paper is to obtain a theorem dealing with the absolute matrix summability of infinite series. A positive sequence (γ_n) is said to be a quasi β-power increasing sequence if there exists a constant $K = K(\beta, \gamma) \geq 1$ such that $Kn^{\beta}\gamma_n \geq m^{\beta}\gamma_m$ holds for all $n \geq m \geq 1$ (see [6]). It should be noted that every almost increasing sequence is a quasi β-power increasing sequence for any nonnegative β, but the converse need not be true as can be seen by taking the example, say $\gamma_n = n^{-\beta}$ for $\beta > 0$. A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty$.

Now, we prove the following result dealing with absolute matrix summability.

Theorem 3.1
Let (X_n) be quasi β-power increasing sequence for some $0 < \beta < 1$ and $A = (a_{nv})$ be a positive normal matrix such that

$$\bar{a}_{n0} = 1, \qquad n = 0, 1, \ldots, \tag{7}$$

$$a_{n-1,v} \geq a_{nv} \qquad for\ n \geq v + 1, \tag{8}$$

$$a_{nn} = O\Big(\frac{p_n}{P_n}\Big), \tag{9}$$

$$\sum_{v=1}^{n-1} \frac{|\hat{a}_{n,v+1}|}{v} = O(a_{nn}). \tag{10}$$

Let $\left(\frac{\varphi_n p_n}{P_n}\right)$ *be a non-increasing sequence and* $(\lambda_n) \in \mathcal{BV}$*. If conditions* (1)–(2) *of Theorem 2.1 and*

$$\sum_{n=1}^{m} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} \frac{|t_n|^k}{n} = O(X_m) \quad as \quad m \to \infty, \tag{11}$$

$$\sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^{k} |t_n|^k = O(X_m) \quad as \quad m \to \infty, \tag{12}$$

are satisfied, then the series $\sum a_n \lambda_n$ *is summable* $\varphi - |A, p_n|_k$*,* $k \geq 1$.

Taking (X_n) as an almost increasing sequence, $\varphi_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 3.1, we get Theorem 2.1.

Lemma 3.2 ([3])
If (X_n) *is quasi* β*-power increasing sequence for some* $0 < \beta < 1$*, then under the conditions* (1) *and* (2)*, we have*

$$nX_n |\Delta \lambda_n| = O(1) \quad as \quad n \to \infty, \tag{13}$$

$$\sum_{n=1}^{\infty} X_n |\Delta \lambda_n| < \infty. \tag{14}$$

4. Proof of Theorem 3.1

Proof of Theorem 3.1. Let (M_n) denotes A-transform of the series $\sum a_n \lambda_n$. We get by (5) and (6),

$$\bar{\Delta} M_n = \sum_{v=0}^{n} \hat{a}_{nv} a_v \lambda_v = \sum_{v=1}^{n} \frac{\hat{a}_{nv} \lambda_v}{v} v a_v.$$

Then, using Abel's transformation, we obtain

$$\begin{aligned}
\bar{\Delta} M_n &= \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv} \lambda_v}{v}\right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn} \lambda_n}{n} \sum_{r=1}^{n} r a_r \\
&= \sum_{v=1}^{n-1} \frac{v+1}{v} \Delta_v (\hat{a}_{nv}) \lambda_v t_v + \sum_{v=1}^{n-1} \frac{v+1}{v} \hat{a}_{n,v+1} \Delta \lambda_v t_v \\
&\quad + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \lambda_{v+1} \frac{t_v}{v} + \frac{n+1}{n} a_{nn} \lambda_n t_n \\
&= M_{n,1} + M_{n,2} + M_{n,3} + M_{n,4}.
\end{aligned}$$

For the proof of Theorem 3.1, we prove

$$\sum_{n=1}^{\infty} \varphi_n^{k-1} |M_{n,r}|^k < \infty \qquad \text{for } r = 1, 2, 3, 4.$$

For $r=1$, applying Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1$, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}|M_{n,1}|^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v||t_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|^{\frac{1}{k}}|\Delta_v(\hat{a}_{nv})|^{\frac{k-1}{k}}|\lambda_v||t_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\left(\sum_{v=1}^{n-1}\left(|\Delta_v(\hat{a}_{nv})|^{\frac{1}{k}}|\lambda_v||t_v|\right)^k\right)^{\frac{1}{k}}\right)^k\\ &\quad\times\left(\left(\sum_{v=1}^{n-1}\left(|\Delta_v(\hat{a}_{nv})|^{\frac{k-1}{k}}\right)^{k'}\right)^{\frac{1}{k'}}\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v|^k|t_v|^k\right)\\ &\quad\times\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}.\end{aligned}$$

Using (4) and (3), we can easily show that $\Delta_v(\hat{a}_{nv})=a_{nv}-a_{n-1,v}$. Then, by (8), (3) and (7), we have

$$\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|=\sum_{v=1}^{n-1}(a_{n-1,v}-a_{nv})\le a_{nn}.$$

Now, (9) gives

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}|M_{n,1}|^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v|^k|t_v|^k\right)\\ &= O(1)\sum_{v=1}^{m}|\lambda_v|^k|t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\Delta_v(\hat{a}_{nv})|\\ &= O(1)\sum_{v=1}^{m}|\lambda_v|^k|t_v|^k\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}\sum_{n=v+1}^{m+1}|\Delta_v(\hat{a}_{nv})|.\end{aligned}$$

Here, (8) implies that

$$\sum_{n=v+1}^{m+1}|\Delta_v(\hat{a}_{nv})|=\sum_{n=v+1}^{m+1}(a_{n-1,v}-a_{nv})\le a_{vv},$$

so, by using the conditions (9), (12), (14) and (1), we obtain

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}|M_{n,1}|^k &= O(1)\sum_{v=1}^{m}|\lambda_v|^{k-1}|\lambda_v||t_v|^k\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}a_{vv}\\ &= O(1)\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^k|\lambda_v||t_v|^k\\ &= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|\sum_{r=1}^{v}\varphi_r^{k-1}\left(\frac{p_r}{P_r}\right)^k|t_r|^k\\ &\quad+O(1)|\lambda_m|\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^k|t_v|^k\\ &= O(1)\sum_{v=1}^{m-1}|\Delta\lambda_v|X_v+O(1)|\lambda_m|X_m\\ &= O(1)\quad\text{as}\quad m\to\infty.\end{aligned}$$

For $r=2$, again using Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1$, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}|M_{n,2}|^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}(|\hat{a}_{n,v+1}||\Delta\lambda_v|)^{\frac{1}{k}}(|\hat{a}_{n,v+1}||\Delta\lambda_v|)^{\frac{k-1}{k}}|t_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\left(\sum_{v=1}^{n-1}((|\hat{a}_{n,v+1}||\Delta\lambda_v|)^{\frac{1}{k}}|t_v|)^k\right)^{\frac{1}{k}}\right)^k\\ &\quad\times\left(\left(\sum_{v=1}^{n-1}((|\hat{a}_{n,v+1}||\Delta\lambda_v|)^{\frac{k-1}{k}})^{k'}\right)^{\frac{1}{k'}}\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|^k\right)\\ &\quad\times\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v|\right)^{k-1}.\end{aligned}$$

Here, by (4), (3) and (8),

$$\begin{aligned}\hat{a}_{n,v+1}=\bar{a}_{n,v+1}-\bar{a}_{n-1,v+1} &= \sum_{i=v+1}^{n}a_{ni}-\sum_{i=v+1}^{n-1}a_{n-1,i}\\ &= a_{nn}+\sum_{i=v+1}^{n-1}(a_{ni}-a_{n-1,i})\le a_{nn}.\end{aligned}$$

Then considering the fact that $(\lambda_n) \in \mathcal{BV}$ and the condition (9), we have

$$\begin{aligned}\sum_{n=2}^{m+1} \varphi_n^{k-1}|M_{n,2}|^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|^k\right)\\ &= O(1)\sum_{v=1}^{m}|\Delta\lambda_v||t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\hat{a}_{n,v+1}|\\ &= O(1)\sum_{v=1}^{m}|\Delta\lambda_v||t_v|^k\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|.\end{aligned}$$

Hence, by (4), (3), (7) and (8), it is clear that $|\hat{a}_{n,v+1}| = \sum_{i=0}^{v}(a_{n-1,i} - a_{ni})$. Then, in view of (3) and (7) we get

$$\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}| = \sum_{n=v+1}^{m+1}\sum_{i=0}^{v}(a_{n-1,i} - a_{ni}) \leq 1. \tag{15}$$

Therefore, by using Abel's transformation, and (11), (2), (14), (13) we obtain

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}|M_{n,2}|^k &= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} v|\Delta\lambda_v|\frac{|t_v|^k}{v}\\ &= O(1)\sum_{v=1}^{m-1}\Delta(v|\Delta\lambda_v|)\sum_{r=1}^{v}\left(\frac{\varphi_r p_r}{P_r}\right)^{k-1}\frac{|t_r|^k}{r}\\ &\quad + O(1)m|\Delta\lambda_m|\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}\frac{|t_v|^k}{v}\\ &= O(1)\sum_{v=1}^{m-1} vX_v|\Delta^2\lambda_v| + O(1)\sum_{v=1}^{m-1}|\Delta\lambda_v|X_v\\ &\quad + O(1)m|\Delta\lambda_m|X_m\\ &= O(1) \quad \text{as} \quad m\to\infty.\end{aligned}$$

Now, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}|M_{n,3}|^k &\leq \sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|}{v}\right)^k\\ &\leq \sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}|\lambda_{v+1}|^k|t_v|^k\right)\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}\right)^{k-1}\\ &= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}\right)\\ &= O(1)\sum_{v=1}^{m}|\lambda_{v+1}|^{k-1}|\lambda_{v+1}|\frac{|t_v|^k}{v}\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\hat{a}_{n,v+1}|\end{aligned}$$

$$= O(1) \sum_{v=1}^{m} |\lambda_{v+1}| \frac{|t_v|^k}{v} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} \sum_{n=v+1}^{m+1} |\hat{a}_{n,v+1}|$$

$$= O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} |\lambda_{v+1}| \frac{|t_v|^k}{v}$$

$$= O(1) \sum_{v=1}^{m-1} \Delta|\lambda_{v+1}| \sum_{r=1}^{v} \left(\frac{\varphi_r p_r}{P_r}\right)^{k-1} \frac{|t_r|^k}{r}$$

$$+ O(1)|\lambda_{m+1}| \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} \frac{|t_v|^k}{v}$$

$$= O(1) \sum_{v=1}^{m-1} |\Delta\lambda_{v+1}| X_{v+1} + O(1)|\lambda_{m+1}| X_{m+1}$$

$$= O(1) \quad \text{as} \quad m \to \infty,$$

by (10), (9), (15), (11), (14) and (1). Finally,

$$\sum_{n=1}^{m} \varphi_n^{k-1} |M_{n,4}|^k = O(1) \sum_{n=1}^{m} \varphi_n^{k-1} a_{nn}^k |\lambda_n|^k |t_n|^k$$

$$= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |\lambda_n|^{k-1} |\lambda_n| |t_n|^k$$

$$= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |\lambda_n| |t_n|^k$$

$$= O(1) \quad \text{as} \quad m \to \infty,$$

as in $M_{n,1}$. This completes the proof.

If we take (X_n) as an almost increasing sequence and $\varphi_n = \frac{P_n}{p_n}$, then we get a theorem dealing with $|A, p_n|_k$ summability (see [10]). Also, if we take (X_n) as a positive non-decreasing sequence, $\varphi_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n, then we get a theorem dealing with $|C, 1|_k$ summability (see [8]).

References

[1] Bari, Nina Karlovna, and Sergey B. Stečkin. "Best approximations and differential properties of two conjugate functions." *Trudy Moskov. Mat. Obšč.* 5 (1956): 483-522. Cited on 96.

[2] Bor, Hüseyin. "On two summability methods." *Math. Proc. Cambridge Philos. Soc.* 97, no. 1 (1985): 147-149. Cited on 96.

[3] Bor, Hüseyin, and Lokenath Debnath. "Quasi β-power increasing sequences." *Int. J. Math. Math. Sci.* no. 41-44 (2004): 2371-2376. Cited on 98.

[4] Flett, Thomas Muirhead. "On an extension of absolute summability and some theorems of Littlewood and Paley." *Proc. London Math. Soc. (3)* 7 (1957): 113-141. Cited on 96.

[5] Hardy, Godfrey Harold. *Divergent Series.* Oxford: Oxford University Press, 1949. Cited on 95.

[6] Leindler, László. "A new application of quasi power increasing sequences." *Publ. Math. Debrecen* 58, no. 4 (2001): 791-796. Cited on 97.

[7] Maddox, Ivor John. *Introductory Mathematical Analysis.* Bristol: Adam Hilger Ltd., 1977. Cited on 95.

[8] Mazhar, Syed Mohammad. "On $|C, 1|_k$ summability factors of infinite series." *Indian J. Math.* 14 (1972): 45-48. Cited on 102.

[9] Mazhar, Syed Mohammad. "Absolute summability factors of infinite series." *Kyungpook Math. J.* 39, no. 1 (1999): 67-73. Cited on 96.

[10] Öğdük, H. Nedret. "A summability factor theorem by using an almost increasing sequence." *J. Comput. Anal. Appl.* 11, no. 1 (2009): 45-53. Cited on 97 and 102.

[11] Özarslan, Hikmet Seyhan, and Enes Yavuz. "A new note on absolute matrix summability." *J. Inequal. Appl.* (2013): Article 474. Cited on 97.

[12] Özarslan, Hikmet Seyhan, and Ayşegül Keten. "On a new application of almost increasing sequences." *J. Inequal. Appl.* (2013): Article 13. Cited on 97.

[13] Özarslan, Hikmet Seyhan, and Enes Yavuz. "New theorems for absolute matrix summability factors." *Gen. Math. Notes* 23, no. 2 (2014): 63-70. Cited on 97.

[14] Özarslan, Hikmet Seyhan. "On generalized absolute matrix summability methods." *Int. J. Anal. Appl.* 12, no. 1 (2016): 66-70. Cited on 96.

[15] Özarslan, Hikmet Seyhan, and Ahmet Karakaş. "A new result on the almost increasing sequences." *J. Comput. Anal. Appl.* 22, no. 6 (2017): 989-998. Cited on 97.

[16] Özarslan, Hikmet Seyhan, and Bağdagül Kartal. "A generalization of a theorem of Bor." *J. Inequal. Appl.* (2017): Article 179. Cited on 97.

[17] Özarslan, Hikmet Seyhan. "A new application of quasi power increasing sequences." *AIP Conference Proceedings* 1926, (2018): 1-6. Cited on 97.

[18] Özarslan, Hikmet Seyhan. "A new factor theorem for absolute matrix summability." *Quaest. Math.* 42, no. 6 (2019): 803-809. Cited on 97.

[19] Özarslan, Hikmet Seyhan. "Generalized quasi power increasing sequences." *Appl. Math. E-Notes* 19 (2019): 38-45. Cited on 97.

[20] Özarslan, Hikmet Seyhan. "An application of absolute matrix summability using almost increasing and δ-quasi-monotone sequences." *Kyungpook Math. J.* 59, no. 2 (2019): 233-240. Cited on 97.

[21] Sulaiman, Waadallah Tawfeeq. "Inclusion theorems for absolute matrix summability methods of an infinite series. IV." *Indian J. Pure Appl. Math.* 34, no. 11 (2003): 1547-1557. Cited on 96.

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Received: April 11, 2019; final version: October 21, 2019;
available online: February 7, 2020.