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Report of Meeting

Conference on Ulam's Type Stability Rytro, Poland, June 2-6, 2014

Contents

The Conference on Ulam's Type Stability (CUTS) was held in Rytro (Poland) on June 2-6, 2014. It was organized by the Department of Mathematics of the Pedagogical University of Cracow. The meeting was focused on various investigations motivated by the notion of Hyers-Ulam stability.

The Organizing Committee of the CUTS consisted of Janusz Brzdęk (chairman), Krzysztof Ciepliński (co-chairman), Anna Bahyrycz (vice-chairman), Magdalena Piszczek (vice-chairman), Zbigniew Leśniak (scientific secretary), Jolanta Olko (scientific secretary), and Paweł Solarz (technical assistance).

The Scientific Committee consisted of Professors: Janusz Brzdęk as chairman, Ravi P. Agarwal, Roman Badora, Krzysztof Ciepliński, Valeriĭ Faĭziev, Michal Fečkan, Soon-Mo Jung, Zbigniew Leśniak, Takeshi Miura, Mohammad Sal Moslehian, Zsolt Páles, Dorian Popa, Themistocles M. Rassias, Prasanna Sahoo, Jens Schwaiger, Marta Štefánková, Jacek Tabor and Bing Xu.

The 60 participants came from 16 countries: Austria, China, Czech Republic, Finland, France, Germany, Hungary, Iran, Morocco, Romania, Serbia, Slovakia, Slovenia, Turkey, United Arab Emirates and Poland.

The conference was opened on Monday, June 2 by Professor Janusz Brzdęk, who welcomed the participants on behalf of the Organizing Committee. The opening address was given by Professor Jacek Chmieliński, the Head of the Department of Mathematics of the Pedagogical University.

During 18 scientific sessions 3 plenary lectures, 3 invited lectures and 46 talks were delivered. They focused mainly on various aspects of Ulam's type stability. Most of the talks concerned functional equations, however differential, integral and integro-differential equations were also discussed as well as some other issues. The participants presented several methods (e.g., direct, fixed point, invariant means, separation and multivalued) for proving stability, and some results on superstability and hyperstability of functional equations. Moreover, some of them dealt with the notion of convexity. On Wednesday, June 4, the special session on discrete dynamical systems was organized by Professor Marta Štefánková from the Mathematical Institute of the Silesian University in Opava. Several contributions have been also made during Problems and Remarks sessions.

On Tuesday, June 3, a regional feast was organized and on the next day afternoon the participants visited Nowy Sącz and Stary Sącz, ones of the oldest towns in Poland. On Thursday, June 5, a farewell dinner was held.

The conference was closed on Friday, June 6, by Professor Janusz Brzdęk who expressed his hope to continue stability meetings in the future.

Abstracts of Talks

Zayid Abdulhadi *On the product combination of logharmonic mappings*

In this paper, we consider the class of logharmonic mappings defined on the unit disc U. A local and global representation will be given. Topics will be included, mapping theorems, logharmonic automorphisms, univalent starlike logharmonic mappings. We obtain some sufficient conditions for the product combination of univalent logharmonic mappings to be starlike. A number of explanatory examples are included.

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Marcin Adam *Stability of the polynomial functional equation in the class of differentiable functions*

During the 15th International Conference on Functional Equations and Inequalities (Ustroń, May 19–25, 2013) we have shown that the class $C^p(\mathbb{R},\mathbb{R})$ of p-times continuously differentiable functions has the difference property of p-th order, i.e. if a function $f\colon \mathbb{R} \to \mathbb{R}$ is such that $\Delta_h^p f \in C^p(\mathbb{R} \times \mathbb{R}, \mathbb{R})$, where $\Delta_h^p f$ is the p-th iterate of the difference operator $\Delta_h f(x) := f(x+h) - f(x)$, then there exists a unique polynomial function $P\colon \mathbb{R} \to \mathbb{R}$ of $(p-1)$-th order such that $f - P \in C^p(\mathbb{R},\mathbb{R})$. Inspired by some results concerning the stability of the Cauchy functional equation (see [1]), as an application of our previous theorem we will present some stability result for the polynomial functional equation in the class of differentiable functions.

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Pekka Alestalo *Hyers-Ulam theorem for bounded sets and applications*

We present a version of the Hyers-Ulam theorem for bounded sets [1]. The result states that for $A \subset \mathbb{R}^n$ bounded, an ε-nearisometry $f\colon A \to \mathbb{R}^n$ can always be approximated by an isometry in such a way that the error term is proportional to $\sqrt{\varepsilon}$, and in many cases even to ε.

These results are then applied to prove extension theorems for $(1+\varepsilon)$-bilipschitz maps $f\colon A \to \mathbb{R}^n$, [2, 3], and more generally, for the so-called power-quasisymmetric maps, [4, 5].

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Szilárd András *Ulam-Hyers stability of integral equations with weak singularities* (joint work with **Árpád Baricz** and **Tibor Pogány**)

The main aim of our talk is to study the Ulam-Hyers stability of some singular integral equations by using fixed point techniques and the theory of Picard operators. Our methods can be used both on timescales for dynamic equations and for integral equations on a half-line. We apply the same methods to study the

Ulam-Hyers stability of the Bessel differential equation and to the hypergeometric differential equation.

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Roman Badora *Invariant means in the theory of stability*

Let $(S,+)$ be a semigroup. For a function f on S with values in a set Y and $a \in S$ the left and the right translations ${}_af$ and f_a be defined by the following formulae

$$ {}_af(x) = f(a+x), \quad f_a(x) = f(x+a), \qquad x \in S. $$

A real linear functional M defined on the space $B(S,\mathbb{R})$ of all real bounded functions on S is called a *left* (*right*) *invariant mean* if and only if it satisfies the following conditions:

$$ \inf_{x \in S} f(x) \le M(f) \le \sup_{x \in S} f(x) $$

and

$$ M({}_af) = M(f) \quad (M(f_a) = M(f)) $$

for all $f \in B(S,\mathbb{R})$ and $a \in S$.

A semigroup S which admits a left (right) invariant mean on the space $B(S,\mathbb{R})$ will be termed *left* (*right*) *amenable*.

The classical result on invariant means (J. von Neumann, J. Dixmier and M.M. Day) states that every commutative semigroup is amenable.

L. Székelyhidi, in 1985, for the first time applied the method of invariant means in the theory of functional equations (L. Székelyhidi, *Remark 17*, Report of Meeting, Aequationes Math. 29 (1985), 95–96). From this time, using the method of invariant means, many interesting results concerning functional equations and inequalities were proved.

In the talk we present applications of invariant means in the stability theory of functional equations.

Anna Bahyrycz *On hyperstability of the general linear equation*
(joint work with **Jolanta Olko**)

We consider the functional equation

$$\sum_{i=1}^{m} A_i g\bigg(\sum_{j=1}^{n} a_{ij}x_j\bigg) + A = 0, \tag{1}$$

with $A, a_{ij} \in \mathbb{F}$, $A_i \in \mathbb{F}\setminus\{0\}$, $i \in \{1,\dots,m\}$, $j \in \{1,\dots,n\}$ for functions g mapping a normed space into a normed space (both over the field $\mathbb{F} \in \{\mathbb{R},\mathbb{C}\}$).

We present a result on hyperstability of the equation (1) and applying it we obtain sufficient condition for the hyperstability of the wide class of functional equations and control functions.

We also show how our outcome may be used to check if the particular functional equation is hyperstable.

Bogdan Batko *Superstability of some conditional functional equations*

We present some superstability results, in the sense of Baker, of selected conditional functional equations with the condition dependent on an unknown function.

Zoltán Boros *Rolewicz theorem for convexity of higher order*
(joint work with **Noémi Nagy**)

Let $I \subset \mathbb{R}$ be an open interval. Rolewicz [2] investigated functions $f\colon I \to \mathbb{R}$ that satisfy inequalities of the form

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) + Ct(1-t)\alpha(|x-y|) \tag{1}$$

for every $x, y \in I$, $t \in [0,1]$, with a non-negative constant C and a nondecreasing function $\alpha\colon [0,+\infty[\to [0,+\infty]$ fulfilling $\lim_{t\to 0+} \frac{\alpha(t)}{t^2} = 0$. He proved that every continuous solution f of inequality (1) has to be convex.

Following Hopf's theses (Berlin, 1926) and Popoviciu's monograph [1], a function $f\colon I \to \mathbb{R}$ is called convex of order n if

$$[x_0, x_1, \dots, x_n, x_{n+1};\, f] \ge 0$$

for all $x_0 < x_1 < \ldots < x_n < x_{n+1}$ in I, where $[x_0, x_1, \dots, x_n, x_{n+1};\, f]$ denotes the divided difference of f at the points x_0, x_1, …, x_n, x_{n+1}.

As a generalized and higher order version of Rolewicz's theorem, we establish the following result.

Theorem
Let $n \in \mathbb{N}$, $\nu(I)$ denote the length of the interval I, and $J_I =]0, \nu(I)[$. Let the function $\varphi\colon J_I \to [0,+\infty[$ satisfy $\lim_{t\to 0+} \varphi(t) = 0$. If a function $f\colon I \to \mathbb{R}$ satisfies the inequality

$$[x_0, x_1, \dots, x_n, x_{n+1};\, f] + \varphi(x_{n+1} - x_0) \ge 0$$

for all $x_0 < x_1 < \ldots < x_n < x_{n+1}$ in I, then f is convex of order n.

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Janusz Brzdęk *On stability of a family of functional equations*
(joint work with **Liviu Cădariu**)

We present some general stability (and hyperstability) results for the functional equation of the following form

$$f(px + r_1y) + f(qx + r_2y) = f(x) + \sum_{i=1}^{m} \alpha_i f(s_iy) + D(x, y),$$

in the class of functions f mapping a commutative group $(X, +)$ into a Banach space Y (over a field $\mathbb{F}$ of real or complex numbers), where the function $D\colon X^2 \to Y$ is given, $m \in \mathbb{N}$, $\alpha_1, \ldots, \alpha_m \in \mathbb{F}$ and $p, q, r_1, r_2, s_1, \ldots, s_m$ are fixed endomorphisms of X.

We also provide some applications of those outcomes for characterizations of inner product spaces and stability of $*$-homomorphisms of C^*-algebras.

The main result has been obtained by a fixed point method.

Liviu Cădariu *Fixed point theory and the Hyers-Ulam stability of functional equations*

The fixed point method is one of the most popular technique extensively used for proving properties of Hyers-Ulam stability of several functional equations. A lot of authors follow the approaches from [2] and [5] and make use of a theorem of Diaz and Margolis.

The aim of this talk is to present applications of some fixed point theorems to the theory of Hyers-Ulam stability for several functional equations.

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Jacek Chmieliński *Orthogonality equations and their stability*

Some stability aspects of the *orthogonality equation*

$$\langle f(x)|f(y)\rangle = \langle x|y\rangle, \qquad x, y \in X$$

and its "pexiderization"

$$\langle f(x)|g(y)\rangle = \langle x|y\rangle, \qquad x, y \in X$$

will be considered. Here X, Y are inner product spaces and $f, g\colon X \to Y$ unknown functions. As for the former equation, the results are surveyed, e.g., in [1].

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Jacek Chudziak *Stability problem for composite type functional equations*

Composite type functional equations play a significant role in various branches of mathematics and they have several interesting applications. Therefore there is an objective need of considering their stability properties. The main difficulty in such studies is caused by the fact that the classical methods of stability theory are, in general, useless in the case of the composite equations.

In this talk we discuss various aspects of the stability problem for the Gołąb-Schinzel equation

$$f(x + f(x)y) = f(x)f(y)$$

and its further generalizations. Moreover we present some results concerning approximate dynamical systems on interval. This problem is closely related to stability of the translation equation.

Krzysztof Ciepliński *On a functional equation characterizing multi-additive mappings and its Hyers-Ulam stability*

Let us recall that a function $f\colon V^n \to W$, where V is a commutative semigroup, W is a linear space and n is a positive integer, is called *multi-additive* or *n-additive* if it is additive (satisfies Cauchy's functional equation) in each variable.

In the talk, we give a functional equation which characterizes multi-additive mappings. Moreover, we present some results on the generalized Hyers-Ulam stability of this equation in normed, non-Archimedean normed and 2-normed spaces.

Marek Czerni *Set stability of Shanholt type for linear functional equations*

G.A. Shanholt gave in his paper [1] the definition of stability of closed sets in finite dimensional normed space in relation to solutions of difference equation.

In this talk we consider stability of Shanholt type of normal regions on the plane with respect to the family of continuous solutions of the linear functional equation of the form

$$\varphi[f(x)] = g(x)\varphi(x) + h(x) \tag{1}$$

with the unknown function φ.

The given functions f, g and h will be subjected to the following conditions:

(H_1) The function f is strictly increasing, continuous mapping from interval $(0,a)$, $0 < a \le \infty$ onto itself and $0 < f(x) < x$ for $x \in (0,a)$.

(H_2) The function g is defined and continuous on the interval $(0,a)$ and $g(x) > 0$ for $x \in (0,a)$.

(H_3) The function h is defined and continuous on the interval $(0,a)$.

It is known that if the given functions f, g and h fulfill hypotheses $(H_1) - (H_3)$, then the continuous solutions of equation (1) depend on an arbitray function.

We shall consider also a particular case of this type of stability and show that such a stability implies the stability in the sense of Hyers-Ulam of equation (1) in some special class of functions.

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Jana Dvořáková *Chaos in nonautonomous discrete dynamical systems*

We consider nonautonomous discrete dynamical systems $(I, f_{1,\infty})$ given by sequences $\{f_n\}_{n\ge1}$ of surjective continuous maps $f_n\colon I \to I$ converging uniformly to a map $f\colon I \to I$ and study some aspects of chaotic behavior of such systems. Recently it was proved, among others, that generally there is no connection between chaotic behavior of $(I, f_{1,\infty})$ and chaotic behavior of the limit function f. We show that even the full Lebesgue measure of a distributionally scrambled set of the nonautonomous system does not guarantee the existence of distributional chaos of the limit map and conversely, that there is a nonautonomous system with arbitrarily small distributionally scrambled set that converges to a map distributionally chaotic a.e.

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Nasrin Eghbali *An approach to the stability of fractional differential equations*

Let D^α be the Caputa fractional derivative, $\beta > 0$, $f\colon J \times C([-h,0],\mathbb{R}) \to \mathbb{R}$, $(J = [t_0,\infty))$, be a given function satisfying some assumptions that will be specified, $h > 0$ and $\phi \in C([t_0 - h, t_0],\mathbb{R})$. Consider the following fractional differential equation

$$\begin{aligned} D^\alpha[y(t)e^{\beta t}] &= f(t,y(t))e^{\beta t}, \qquad t \in [t_0,\infty),\ t_0 \ge 0,\ 0 < \alpha < 1, \\ y(t) &= \phi(t), \qquad t_0 - h \le t \le t_0. \end{aligned} \tag{1}$$

Let $y \in C([t_0 - h, \infty), \mathbb{R})$. For every $t \in [t_0, \infty)$, define y_t by

$$y_t(\theta) = y(t + \theta), \qquad \theta \in [-h, 0].$$

Denote by $BC([t_0 - h, \infty), \mathbb{R})$ the Banach space of all bounded continuous functions from $[t_0 - h, \infty)$ into $\mathbb{R}$ with the norm $\|y\|_\infty = \sup\{|y(t)| : \ t \in [t_0 - h, \infty)\}$. Assume that $f(t, x_t)$ is Lebesgue measurable with respect to t on $[t_0, \infty)$, and $f(t_0, \phi)$ is continuous with respect to ϕ on $C([-h, 0], \mathbb{R})$. By the technique used in [1], we get the equivalent form of the equation (1) as

$$\begin{aligned} &y(t) = y(t_0)e^{-\beta(t-t_0)} + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} e^{-\beta(t-s)} f(s, y_s)\, ds, \qquad t \geq t_0, \\ &y(t) = \phi(t), \qquad t \in [t_0 - h, t_0]. \end{aligned} \tag{2}$$

The formal definition of the Hyers-Ulam-Rassias stability for the equation (2) can be defined as follows:
For each function y satisfying

$$\left| y(t) - y(t_0)e^{-\beta(t-t_0)} - \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} e^{-\beta(t-s)} f(s, y_s)\, ds \right| \leq \psi(t)$$

in which ψ is a nonnegative function, there is a solution y_0 of the equation (2) and a constant $K > 0$ independent of y and y_0 such that

$$|y(t) - y_0(t)| \leq K\psi(t).$$

In this talk we present some results on the stability of the equation (2).

References

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Ajda Fošner *Some results on the stability of functional equations*
(joint work with **Maja Fošner**)

A classical question in the theory of functional equations is: *Under what conditions is it true that a map which approximately satisfies a functional equation $\mathcal{E}$ must be somehow close to an exact solution of $\mathcal{E}$*? We say that a functional equation $\mathcal{E}$ is stable if any approximate solution of $\mathcal{E}$ is near to a true solution of $\mathcal{E}$. We will present some new results on the stability of functional equations. In particular, we will present the stability and hyperstability of cubic Lie derivations on normed algebras.

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Attila Gilányi *On the stability of monomial functional equations*

Let X and Y be linear normed spaces, and let n be a positive integer. A function $f: X \to Y$ is called a monomial function of degree n if it satisfies the functional equation

$$\Delta_y^n f(x) - n! f(y) = 0, \qquad x, y \in X,$$

where Δ denotes the (forward) difference operator. It is well-known and easy to see that this class of equations contains the Cauchy equation and the square-norm (or parallelogram or Jordan–von Neumann or quadratic) equation as a special case. The stability of equations belonging to the class have been investigated by several authors. In this talk, we give a short summary of their results and we present some conditional stability theorems for monomial equations.

Ioan Goleţ *Hyers-Ulam-Rassias stability in fuzzy normed spaces*
(joint work with **Liviu Cădariu**)

The concept of fuzzy sets was introduced by Zadeh [5] in 1965. Since then, many authors have expansively developed the theory of fuzzy sets. Kramosil and Michalek [4] have introduced the concept of fuzzy metric spaces. George and Veeramani [1] modified this concept of fuzzy metric spaces and defined topologies induced by fuzzy metrics. Many authors have proved fixed point theorems in fuzzy metric spaces.

In the same time fuzzy norms have given appropriate generalizations of deterministic normed spaces and have opened the way for new applications.

Recently, considerable attention has given to the Hyers-Ulam-Rassias stability of functional or differential equations. This study is encouraged by the particular applications in the information theory, computer science etc. The fuzzy normed spaces has also offered a proper framework for the study of Hyers-Ulam-Rassias stability of some classical functional equations.

In this paper we use a fixed point result in fuzzy normed space as a tool for investigating the Hyers-Ulam-Rassias stability of a quadratic functional equation.

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Eszter Gselmann *Characterization of derivations through their actions on certain elementary functions*

The main aim of this talk is to provide characterization theorems concerning real derivations.

We say that an additive function $f: \mathbb{R} \to \mathbb{R}$ is a *derivation* if

$$f(xy) = xf(y) + yf(x)$$

is fulfilled for all $x, y \in \mathbb{R}$.

The additive function $f: \mathbb{R} \to \mathbb{R}$ is termed to be a *linear function* if f is of the form

$$f(x) = f(1) \cdot x, \qquad x \in \mathbb{R}.$$

This talk will be devoted to answering affirmatively the following problem in some cases. Assume that $\xi: \mathbb{R} \to \mathbb{R}$ is a given differentiable function and for the additive function $d: \mathbb{R} \to \mathbb{R}$, the mapping

$$x \longmapsto d(\xi(x)) - \xi'(x)d(x)$$

is regular (e.g. locally bounded, measurable, continuous at a point) on its domain. So it true that in this case d admits a representation

$$d(x) = \chi(x) + d(1) \cdot x, \qquad x \in \mathbb{R},$$

where $\chi: \mathbb{R} \to \mathbb{R}$ is a real derivation?

Eliza Jabłońska *Fixed points almost everywhere and Hyers-Ulam stability*
(joint work with **Janusz Brzdęk**)

In 2011 J. Brzdęk, J. Chudziak, Z. Pales [1] and J. Brzdęk, K. Ciepliński [2] proved fixed point theorems for some operators and derived from it several results on the stability of a very wide class of functional equations in single variable. In 2012 L. Cădariu, L. Găvruţa, P. Găvruţa [3] generalized these results.

Here we generalize results from [3]; more precisely, we prove a fixed point theorem almost everywhere and apply it to obtain some stability results.

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Wojciech Jabłoński *Stability and completeness*

Many authors examined stability of various functional equations since the time when S.M. Ulam [4] has posed his problem of stability of the equation of a homomorphism. We discuss these aspects of stability which concerns completeness of a target space (cf. [1], [2] and [3]).

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Zdeněk Kočan *Dynamics on intervals, graphs and dendrites*
(joint work with **Veronika Kurková** and **Michal Málek**)

Let us consider some properties of discrete dynamical systems such as the existence of an horseshoe, the positivity of topological entropy, the existence of a homoclinic trajectory or the existence of maximal omega-limit sets. We survey the known relations between the properties in the case of interval, graph and dendrite maps. In particular, we point out differences between the dynamics on intervals and on dendrites.

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Zbigniew Leśniak *On stability of an operator type equation of order n*
(joint work with **Janusz Brzdęk** and **Stevo Stević**)

We consider an n-th order linear operator type equation with constant coefficients

$$p_n\mathcal{L}^n\psi - p_{n-1}\mathcal{L}^{n-1}\psi + \ldots + (-1)^{n-1}p_1\mathcal{L}\psi + (-1)^n\psi = \theta,$$

where $\mathcal{L}\colon X \to X$, X is a complete extended normed vector space and θ is the zero of X. We show that, under suitable assumptions, the equation is stable in the Hyers-Ulam sense. To obtain this result we use the Diaz-Margolis fixed point theorem.

Renata Malejki *On stability of a generalization of the Fréchet equation*
(joint work with **Anna Bahyrycz**)

We present some stability and hyperstability results for the functional equation

$$A_1f(x+y+z)+A_2f(x)+A_3f(y)+A_4f(z) = A_5f(x+y)+A_6f(x+z)+A_7f(y+z),$$

which is a generalization of the Fréchet equation ($A_1 = \ldots = A_7 = 1$) stemming from one of the characterizations of the inner product spaces. As the main tool in the proofs we have used a fixed point theorem for the function spaces.

Alpár Richárd Mészáros *Ulam-Hyers stability of elliptic PDEs in Sobolev spaces*
(joint work with **Szilárd András**)

In this talk we analyse the Ulam-Hyers stability of some elliptic partial differential equations on bounded domains with Lipschitz boundary. We use direct techniques and also some abstract methods of Picard operators.

The novelty of our approach consists in the fact that we are working in Sobolev spaces and we do not need to know the explicit solutions of the problems or the Green functions of the elliptic operators. We show that the Ulam-Hyers stability of linear elliptic problems do not say much information in plus, it mainly follows from standard estimations for elliptic PDEs, Cauchy-Schwartz and Poincaré type inequalities and Lax-Milgram type theorems.

We obtain powerful results in the sense that working in Sobolev spaces, we can control also the derivatives of the solutions, instead of the known point-wise stability for the moment (see [2]). Moreover, our results for the nonlinear problems generalize in some sense some recent results from the literature (see for example [3]).

These results can be found in our recent paper [1].

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Tímea Nagy *Second order non-linear ODEs with non-local initial conditions. Existence and Ulam-Hyers stability*
(joint work with **Szilárd András**)

In this talk our aim is to study some non-linear second order ODE systems with non-local initial conditions. The model problem is the following system

$$x''(t) = -f_1(t, x(t), y(t)), \quad y''(t) = -f_2(t, x(t), y(t)), \qquad t \in I,$$

$$x(0) = \alpha_0[x], \quad x'(0) = \alpha_1[x], \quad y(0) = \beta_0[y], \quad y'(0) = \beta_1[y],$$

where $\alpha_i, \beta_i\colon \mathcal{C}[0,1] \to \mathbb{R}$, $i \in \{0,1\}$ are linear and continuous operators. In the model $I \subset \mathbb{R}$ is a bounded interval and $f_1, f_2\colon I \times \mathbb{R}^2 \to \mathbb{R}$ are Carathéodory functions. Under certain assumptions on the data functions we are able to show existence of the solutions and Ulam-Hyers stability of the above system. For this analysis we use some fixed point techniques and the core of the approaches relies on vector valued metrics and converging to zero matrices. These techniques were first used for first order non-linear systems with non-local initial conditions (see [1, 2]) and the works [3, 4] motivate this type of study for second order cases.

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Kazimierz Nikodem *Strong convexity and separation theorems*

Let $D \subset \mathbb{R}^n$ be a convex set and c be a positive number. A function $f\colon D \to \mathbb{R}$ is called:
– *strongly convex with modulus* c if

$$f(tx+(1-t)y) \le tf(x)+(1-t)f(y)-ct(1-t)\|x-y\|^2, \quad x,y \in D,\ t \in [0,1];$$

– *approximately concave with modulus* c if

$$f(tx+(1-t)y) \ge tf(x)+(1-t)f(y)-ct(1-t)\|x-y\|^2, \quad x,y \in D,\ t \in [0,1];$$

– *c-quadratic* if

$$f(tx+(1-t)y) = tf(x)+(1-t)f(y)-ct(1-t)\|x-y\|^2, \quad x,y \in D,\ t \in [0,1].$$

A relationship between the above notions (for $n = 1$) and the generalized convexity in the sense of Beckenbach is shown. Characterizations of pairs of functions that can be separated by strongly convex, approximately concave and c-quadratic functions are given. As corollaries some Hyers-Ulam stability results for the above classes of functions are obtained.

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Andrzej Olbryś *On separation by h-convex functions*

Let X will be a real linear space, and let $D \subset X$ be an open and convex set. The concept of h-convexity was introduced by Varošanec [2] in the following way:

Definition
Let $J \subset \mathbb{R}$ be an interval, $(0,1) \subset J$ and let $h\colon J \to \mathbb{R}$ be a non-negative function. We say that $f\colon D \to \mathbb{R}$ is an h-convex function, if f is non-negative and for all $x, y \in D$ and $s \in (0,1)$ we have

$$f(sx + (1-s)y) \leq h(s)f(x) + h(1-s)f(y).$$

In our talk we establish the necessary and sufficient conditions under which two functions can be separated by h-convex function, in the case, where the function h is super-multiplicative. This result is related to the theorem on separation by convex functions presented in [1]. As a consequence of our main theorem we obtain the stability result for h-convex functions.

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Jolanta Olko *On stability of the general linear equation*
(joint work with **Anna Bahyrycz**)

Consider the general linear functional equation of the form

$$\sum_{i=1}^{m} A_i f\left(\sum_{j=1}^{n} a_{ij} x_j\right) + A = 0,$$

where $A, a_{ij} \in \mathbb{F}$, $A_i \in \mathbb{F} \setminus \{0\}$, $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$ in the class of functions mapping a normed space into a Banach space (both over the same field $\mathbb{R}$ or $\mathbb{C}$).

We give sufficient conditions for the generalized Hyers-Ulam stability of the equation. Moreover, we present some applications to particular functional equations of this type and different control functions.

Lahbib Oubbi *Ulam stability of an equation with several parameters*

We deal with the Ulam-Hyers stability of the functional equation

$$\sum_{i=1}^{m} f(a_i x_0 + b_i x_i) + f\left(x_0 - \sum_{i=1}^{m} b_i x_i\right) - f(x_0) = 0, \tag{1}$$

where $m \geq 2$ is an integer, $(a_i)_{i=1,\dots,m}$ and $(b_i)_{i=1,\dots,m}$ are scalars so that $\sum_{i=1}^{m} a_i = 0$ and $b_i \neq 0$ for every $i = 1, \dots, m$. We first show that a mapping f satisfies (1) if and only if it is additive. We then show the Ulam-Hyers-Găvruţa stability of the equation (1). Furthermore, using the classical fixed point theorem, we establish the Ulam-Hyers-Cădariu-Radu stability of (1). As corollary, we get the Ulam-Hyers-Rassias stability of (1). Finally, whenever the considered spaces are algebras, we combine (1) with some other equations such as $f(xy) = f(x)f(y)$ or $f(xy) = \alpha_1 x f(y) + \alpha_2 f(x) y$ and get the stability of different types of mappings (among which the ring derivations and the ring homomorphisms) with respect to the obtained systems of equations. The results obtained here are natural generalizations of the ones obtained in [1] and [2].

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Zsolt Páles *Stability of generalized monomial functional equations*

Given a groupoid $(X, \diamond)$, we define the powers of an element $x \in X$ by

$$x^1 := x, \qquad x^{n+1} := x \diamond x^n, \quad n \in \mathbb{N}.$$

The groupoid as well as the operation $\diamond$ is called power associative if, for all $x \in X$, for all $n, k \in \mathbb{N}$,

$$x^{n+k} = x^n \diamond x^k.$$

Given $\ell \geq 2$, the groupoid as well as the operation $\diamond$ is said to be ℓ-power symmetric if, for all $x, y \in X$,

$$(x \diamond y)^\ell = x^\ell \diamond y^\ell.$$

Obviously, commutative semigroups are power associative and ℓ-power symmetric for every $\ell \geq 2$. On the other hand, there exist non-associative and non-commutative structures that are still power associative and 2-power symmetric.

The main result concerns the Hyers-Ulam stability of the generalized monomial functional equation

$$p_0 f(x) + p_1 f(x \diamond y) + \ldots + p_n f(x \diamond y^n) = f(y), \qquad x, y \in X,$$

where $(G, \diamond)$ is a power associative and ℓ-power symmetric groupoid, $f \colon X \to Y$, Y is a Banach space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ and $p_0, p_1, \dots, p_n \in \mathbb{K}$ with $p_0 + p_1 + \ldots + p_n = 0$.

The particular case when $p_j = \frac{(-1)^{n-j}}{n!}\binom{n}{j}$ for $j \in \{0, 1, \dots, n\}$ is the standard monomial equation which was considered by A. Gilányi in 1999 in the paper [1].

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Paweł Pasteczka *On negative results concerning Hardy means*

A mean $M\colon\bigcup_{n=1}^{\infty}\mathbb{R}_+^n \to \mathbb{R}_+$ is called *Hardy* if there exists $C > 0$ such that $\sum_{n=1}^{\infty} M(a_1,\dots,a_n) < C\sum_{n=1}^{\infty} a_n$ for any sequence of arguments $a \in l^1(\mathbb{R}_+)$. We present some sufficient conditions for a mean *not* to be Hardy.

In 2004 Zs. Páles and L.-E. Persson [1] proved that a Gini Mean

$$G_{p,q}(a_1,\dots,a_n) := \begin{cases} \Big(\dfrac{a_1^p+\dots+a_n^p}{a_1^q+\dots+a_n^q}\Big)^{\frac{1}{p-q}}, & \text{if } p \neq q, \\ \exp\Big(\dfrac{a_1^p\ln(a_1)+\dots+a_n^p\ln(a_n)}{a_1^p+\dots+a_n^p}\Big), & \text{if } p = q \end{cases}$$

(i) is Hardy if $\max(p,q) < 1$ and $\min(p,q) \le 0$,

(ii) is not Hardy if $\max(p,q) > 1$ or $\min(p,q) > 0$.

We are going to prove that the condition (i) is also a necessary one.

Moreover, we are going to precisely tell, in the family of Gaussian Product of Power Means, the Hardy means from the non-Hardy ones.

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Magdalena Piszczek *Stability and hyperstability of the Drygas functional equation*
(joint work with **Joanna Szczawińska**)

Let X be a nonempty subset of an Abelian group and Y be a semigroup. We say that a function $f\colon X \to Y$ satisfies the Drygas functional equation on X if

$$f(x+y)+f(x-y) = 2f(x)+f(y)+f(-y)$$

for all $x, y \in X$ such that $-y, x+y, x-y \in X$.

We use the fixed point theorem for functional spaces to obtain the stability and hyperstability result for the Drygas functional equation on a restricted domain.

Dorian Popa *Hyers-Ulam stability of some equations and operators*
(joint work with **Ioan Raşa**)

We present some results concerning generalized Hyers-Ulam stability for the linear differential equation with constant coefficients and also for the linear differential operator with non-constant coefficients in Banach spaces. A characterization of Hyers-Ulam stability of linear operators in Banach spaces was given in [3].

Using this result we investigate the Hyers-Ulam (in)stability of some classical operators from approximation theory. For some of them we obtain the best constant.

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Barbara Przebieracz *Stability of the translation equation*

The aim of this talk is to present results concerning the stability of the translation equation in some classes of functions.

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Sebaheddin Şevgin *Hyers-Ulam stability of an integro-differential equation* (joint work with **Hamdullah Şevli**)

In [1], S.M. Jung applied the fixed point method for proving the Hyers-Ulam-Rassias stability and the Hyers-Ulam stability of a Volterra integral equation of the second kind. In [2], Morales and Rojas studied the Hyers-Ulam-Rassias types of stability of nonlinear, nonhomogeneous Volterra integral equations with delay on finite intervals. Recently in [3], Jung, Şevgin and Şevli proved that if $p\colon I \to \mathbb{R}$, $q\colon I \to \mathbb{R}$, $K\colon I \times I \to \mathbb{R}$ and $\varphi\colon I \to [0, \infty)$ are sufficiently smooth functions and if a continuously differentiable function $u\colon I \to \mathbb{R}$ satisfies the perturbed integro-differential inequality

$$\Big|u'(t) + p(t)u(t) + q(t) + \int_c^t K(t,\tau)u(\tau)\,d\tau\Big| \le \varphi(t)$$

for all $t \in I$, then there exists a unique solution $u_0\colon I \to \mathbb{R}$ of the Volterra integro-differential equation $u'(t) + p(t)u(t) + q(t) + \int_c^t K(t,\tau)u(\tau)\,d\tau = 0$ such that

$$|u(t) - u_0(t)| \le \exp\Big\{-\int_c^t p(\tau)\,d\tau\Big\}\int_t^b \varphi(\xi)\exp\Big\{\int_c^\xi p(\tau)\,d\tau\Big\}\,d\xi$$

for all $t \in I$.

In this study, we will establish the Hyers-Ulam-Rassias stability and the Hyers-Ulam stability of an integro-differential equation.

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Hamdullah Şevli *Hyers-Ulam stability of Volterra integro-differential equations* (joint work with **Sebaheddin Şevgin**)

Let I be either $(-\infty, b]$, $\mathbb{R}$, $[a, \infty)$, or a closed interval $[a, b]$ with $-\infty < a < b < \infty$, let c be a fixed point of I, and let $\varphi\colon I \to [0, \infty)$ be a continuous function. S.M. Jung [1] proved that if a continuous function $u\colon I \to \mathbb{C}$ satisfies the perturbed Volterra integral inequality

$$\Big|u(t) - \int_c^t F(\tau, u(\tau))\,d\tau\Big| \le \varphi(t)$$

for all $t \in I$, then under some additional conditions, there exist a unique continuous function $u_0\colon I \to \mathbb{C}$ and a constant $C > 0$ such that

$$u_0(t) = \int_c^t F(\tau, u_0(\tau))\,d\tau \qquad \text{and} \qquad |u(t) - u_0(t)| \le C\varphi(t)$$

for all $t \in I$. Recently in [2] the authors jointly with S.M. Jung proved that if $p\colon I \to \mathbb{R}$, $q\colon I \to \mathbb{R}$, $K\colon I \times I \to \mathbb{R}$ and $\varphi\colon I \to [0, \infty)$ are sufficiently smooth functions and if a continuously differentiable function $u\colon I \to \mathbb{R}$ satisfies the perturbed integro-differential inequality

$$\Big|u'(t) + p(t)u(t) + q(t) + \int_c^t K(t,\tau)u(\tau)\,d\tau\Big| \le \varphi(t)$$

for all $t \in I$, then there exists a unique solution $u_0: I \to \mathbb{R}$ of the Volterra integro-differential equation

$$u'(t) + p(t)u(t) + q(t) + \int_c^t K(t,\tau)u(\tau)\,d\tau = 0$$

such that

$$|u(t) - u_0(t)| \le \exp\Big\{ -\int_c^t p(\tau)\,d\tau \Big\} \int_t^b \varphi(\xi) \exp\Big\{ \int_c^\xi p(\tau)\,d\tau \Big\}\,d\xi$$

for all $t \in I$.

In this paper, we will apply the fixed point method for proving the Hyers-Ulam-Rassias stability and the Hyers-Ulam stability of a nonlinear Volterra integro-differential equation.

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Stanisław Siudut *Cauchy difference operator in some Orlicz classes*

Let $(G, \cdot, \lambda)$ be a measurable group with a complete, left-invariant and finite measure λ. If φ is a convex φ-function satisfying conditions $\frac{\varphi(u)}{u} \to 0$ as $u \to 0$, $\frac{\varphi(u)}{u} \to \infty$ as $u \to \infty$, $f: G \to \mathbb{R}$ and the Cauchy difference $\mathcal{C}f(x,y) = f(x \cdot y) - f(x) - f(y)$ of f belongs to $\mathcal{L}^{\varphi}_{\lambda\times\lambda}(G \times G, \mathbb{R})$, then there exists unique additive $A: G \to \mathbb{R}$ such that $f - A \in \mathcal{L}^{\varphi}_{\lambda}(G, \mathbb{R})$. Moreover, $\|f - A\|_{\varphi} \le K\|\mathcal{C}f\|_{\varphi}$, where $K = 1$ if $\lambda(G) \ge 1$, $K = 1 + (\lambda(G))^{-1}$ if $\lambda(G) < 1$. Similar result we also obtain without associativity of $\cdot$ but with $f \in \mathcal{L}^{\varphi}_{\lambda}(G, \mathbb{R})$ and with measurability of $\mathcal{C}f$. In this case $A = 0$ and the Cauchy difference $\mathcal{C}: \{f \in L^{\varphi}_{\lambda}(G, \mathbb{R}) |\ \mathcal{C}f \in L^{\varphi}_{\lambda\times\lambda}(G\times G, \mathbb{R})\} \to L^{\varphi}_{\lambda\times\lambda}(G\times G, \mathbb{R})$ is linear continuous and continuously invertible on its image, where $L^{\varphi}_{\lambda}(G, \mathbb{R})$ ($L^{\varphi}_{\lambda\times\lambda}(G \times G, \mathbb{R})$) denotes the space of equivalence classes of functions in $\mathcal{L}^{\varphi}_{\lambda}(G, \mathbb{R})$ ($\mathcal{L}^{\varphi}_{\lambda\times\lambda}(G \times G, \mathbb{R})$). Moreover, $\mathcal{C}$ is compact if and only if the domain of $\mathcal{C}$ has a finite dimension.

Let $(G, \cdot, \lambda)$ be a measurable group with a complete, left-invariant and σ-finite measure λ such that $\lambda(G) = \infty$. If φ is a φ-function, $f: G \to \mathbb{R}$ and $\mathcal{C}f \in \mathcal{L}^{\varphi}_{\lambda\times\lambda}(G \times G, \mathbb{R})$, then there exist a unique additive $A: G \to \mathbb{R}$ which is equal to f λ a.e.

Dorota Śliwińska *Symmetrization and convexity II*
(joint work with **Szymon Wąsowicz**)

In Sz. Wąsowicz's talk the symmetrization method was presented. We use it to prove the Hermite–Hadamard type inequalities for Wright-convex, strongly convex and strongly Wright-convex functions of several variables defined on simplices.

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Jaroslav Smítal *Distributional chaos after twenty years*

Twenty years ago, in [1] there was introduced the notion of distributional chaos for continuous maps of the interval. The notion was later generalized in [2] and [3] to continuous maps of a compact metric space. There appeared many open problems, some of them were already solved. The most important recent result is a proof of the conjecture, that positive topological entropy implies distributional chaos DC2. In the talk we provide a brief survey of history, main properties of distributional chaos, main open problems, and possible directions of other research of the field.

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Marta Štefánková *On the Sharkovsky classification program of triangular maps*

For continuous interval maps there are more than 50 mutually equivalent conditions characterizing maps with zero topological entropy. At the end of the 1980s Sharkovsky proposed to verify which of the implications between these conditions are valid in the class of triangular maps of the unit square. Since some conditions are not applicable to maps of the square whereas some new conditions have been added thereafter the contemporary list usually contains 32 conditions which means nearly 1000 possible implications. This huge program has been recently completed and in my talk I will give a survey of these results with emphasis on the most recent ones.

Stevo Stević *Unique existence of solutions of a class of nonlinear functional equations in a neighborhood of zero*

We present a unique existence result for a solution of the following system of nonlinear functional equations with iterated deviations in a neighborhood of zero and satisfying the Lipschitz condition

$$x(t) = f\big(t, x(v_1^{(1)}(t,x)), \dots, x(v_1^{(k)}(t,x))\big),$$

where

$$v_j^{(i)}(t,x) = \alpha_{ij}t + \varphi_{ij}(t, x(\alpha_{i\,j+1}t + \varphi_{i\,j+1}(\ldots x(\alpha_{i\,m_i}t + \varphi_{i\,m_i}(t, x(t)))\ldots)))$$

$\alpha_{ij} \in \mathbb{R}$, $i = \overline{1,k}$, $j = \overline{1,m_i}$, $\varphi_{ij}(t,x)$, $i = \overline{1,k}$, $j = \overline{1,m_i}$, are real functions, $f(t,x_1,\ldots,x_k)$ is a real vector function satisfying some additional conditions, and $x(t)$ is an unknown vector function on a subset of $\mathbb{R}^N$.

Leszek Szała *Chaotic behavior of discrete dynamical systems with randomly perturbed trajectories*

My talk will concern recurrence for discrete dynamical systems defined on the unit interval I or on the cube I^n with randomly perturbed trajectories. Both autonomous and nonautonomous case will be concerned. A review of known results as well as some of my recent ones, showing relations between classical recurrence (i.e. for systems without random perturbations) and the so called (f,δ)-recurrence will be presented.

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Joanna Szczawińska *Selections of generalized convex set-valued functions with bounded diameter*
(joint work with **Andrzej Smajdor**)

Let $\alpha \in (-1,1)$, $p,q > 0$, K be a subset of a vector space X such that $0 \in K$ and $K \subset pK$ and let $(Y, \|\cdot\|)$ be a real Banach space. Consider a set-valued function $F\colon K \to cl(Y)$ satisfying the following conditional inclusion

$$\alpha F(x) + (1-\alpha)F(y) \subset F(px+qy), \qquad x,y \in K,\ px+qy \in K.$$

We prove that if the set-valued function F has a bounded diameter, i.e.

$$\sup\{\mathrm{diam} F(x),\ x \in K\} = M < +\infty,$$

then there exists a unique function $f\colon K \to Y$ such that

$$\alpha f(x) + (1-\alpha)f(y) = f(px+qy), \qquad x,y \in K,\ px+qy \in K$$

and

$$f(x) + F(0) \subset F(x), \qquad x \in K.$$

We also apply the method used in the proof to the investigation of the stability of the functional equation

$$\alpha f(x) + (1-\alpha)f(y) = f(px+qy).$$

László Székelyhidi *Stability of functional equations on hypergroups*

In this talk we present stability results for diverse functional equations on hypergroups. The functional equations involved characterize exponentials, additive functions and moment functions. We also consider equations of mixed type. The results depend on amenability and superstability, as well as on a combination of these properties.

Tomasz Szostok *Stability of functional equations stemming from numerical analysis*

We study the stability properties of the equation

$$F(y) - F(x) = (y - x)\sum_{i=1}^{n} a_i f(\alpha_i x + \beta_i y) \tag{1}$$

which is motivated by the numerical integration. In [1] the stability of the simplest equation of the type (1) was investigated thus the inequality

$$|F(y) - F(x) - (y - x)f(x + y)| \leq \varepsilon$$

was studied. In the current paper we present a somewhat different approach to the problem of stability of (1). Namely, we deal with the inequality

$$\left|\frac{F(y) - F(x)}{y - x} - \sum_{i=1}^{n} a_i f(\alpha_i x + \beta_i y)\right| \leq \varepsilon.$$

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Jacek Tabor *Stability of the elastic maps*
(joint work with **Józef Tabor** and **Ewa Matczyńska**)

In our talk we study the stability of the *elastic maps*. Roughly speaking [1], a function $e\colon \{-n, \ldots, n\} \times \{-k, \ldots, k\} \to \mathbb{R}^d$ is called an elastic map if it approximately satisfies the Jensen condition

$$e\Big(\frac{p + q}{2}\Big) = \frac{e(p) + e(q)}{2}$$

for $p, q, \frac{p+q}{2} \in dom(e)$ such that $(p, q) \in N(\mathbb{Z}^2)$, where $N(\mathbb{Z}^2)$ denotes the set of neighbors in $\mathbb{Z}^2$, that is points such that $\|p - q\|_{\max} \leq 2$. We also discuss the case when the set of neighbors is exchanged with its subset.

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Szymon Wąsowicz *Symmetrization and convexity I*
(joint work with **Alfred Witkowski**)

It is well-known that the Hermite part of the Hermite–Hadamard inequality estimates the integral mean value of a convex function of one real variable better than the Hadamard part. Unfortunately, this is not the case in the multivariate case. Nevertheless, it is possible to give a refinement going in this spirit. The presented result we prove by using the symmetrization method.

References

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Paweł Wójcik *On a restriction of an operator to an invariant subspace*

For Banach spaces we consider the bounded linear operators which are surjective and noninjective. The aim of this report is to discuss an invariant subspace of some surjective operators. We show some general properties of such mappings. We examine whether such operators can restrict to an involution or a projection. Thus, we will obtain the invariant subspaces for those operators.

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Peter Volkmann *Bounded perturbations of additive functions*

The topic will be discussed within the scope of Pólya-Szegő-Hyers-Ulam stability.

Pavol Zlatoš *Stability of homomorphisms in the compact-open topology*

It seems that so far stability of homomorphisms between topological groups was studied exclusively from the global point of view, related to the topology of uniform convergence, dealing with approximability of everywhere defined maps by continuous homomorphisms. We introduce a local notion of stability, related to the compact-open topology, dealing with approximate extendability of partial maps to continuous homomorphisms.

We prove that the characters of any locally compact abelian group are locally stable in this sense. We also generalize a global stability result on continuous homomorphisms between compact groups to a local version for continuous homomorphisms from any locally compact group to an arbitrary topological group and show the necessity of the assumptions of the theorem through some counterexamples. As an application we obtain a purely algebraic result on extendability of finite partial maps to homomorphisms.

Some of these results can be generalized to homomorphisms between topological universal algebras. By means of Nonstandard Analysis some of them can be strengthened, adding them certain kind of uniformity.

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Problem and Remarks

1. Problem.

Let $(E, \|\cdot\|)$ be a normed space. The function $\|\cdot\|^2$ is called *c-convex* (for some $c \in (0,1]$) if

$$(1-a)\|x\|^2 + a\|y\|^2 - \|(1-a)x + ay\|^2 \geq ca(1-a)\|x-y\|^2$$

for all $x, y \in E$, $a \in [0,1]$.

Let $p \geq 1$, $E = \mathbb{R}^2$, $\|x\|_p = (|x_1|^p + |x_2|^p)^{\frac{1}{p}}$. Then $\|\cdot\|_2^2$ is 1-*convex*. It is not difficult to prove that for $p = 1$ and $p > 2$, $\|x\|_p^2$ is not *c-convex*.

Problem

(i) Prove or disprove that there exists $r \in (1,2)$ such that

(1) If $p \in (1,r)$, $\|\cdot\|_p^2$ is not *c-convex*,

(2) If $p \in (r,2)$, $\|\cdot\|_p^2$ is c_p-*convex*, for some $c_p \in (0,1)$.

(ii) If r exists, estimate r and c_p. What about $\|\cdot\|_r^2$?

Ioan Raşa

2. Problem.

For a given $p \geq 1$ consider the fundamental Bernstein polynomials

$$b_{p,i}(x) := \binom{p}{i} x^i (1-x)^{p-i}, \qquad i = 0, 1, \ldots, p; \; x \in [0,1].$$

Prove or disprove:

For each convex function $f \in \mathcal{C}([0,1])$ and for all $x, y \in [0,1]$,

$$\sum_{i,j=0}^{p} (b_{p,i}(x)b_{p,j}(x) + b_{p,i}(y)b_{p,j}(y) - 2b_{p,i}(x)b_{p,j}(y)) f\left(\frac{i+j}{2p}\right) \geq 0.$$

Ioan Raşa

3. Problem. Let $k(n)$ be defined by

$$k(1) = 0, \; k(2) = 1, \; k(3) = 3,$$
$$k(n) = \min\left\{k \in \mathbb{N} : \sum_{i=1}^{n-1} \binom{n}{i} \left(1 - \frac{i}{n}\right)^k < 1\right\}, \qquad n \geq 4.$$

Theorem

If $f \in C^{k(n)}([0,1])$ is a solution of the equation

$$\Delta_t^n f(0) = 0, \qquad \forall t \in \left[0, \frac{1}{n}\right],$$

then f is polynomial of degree $\leq n-1$.

(See D. Popa, I. Raşa, J. Approx. Theory 164 (2012), 138–144).

It can be proved that $k(n) \leq n^2 \log 2$, $n \geq 4$ and

$$\lim_{n\to\infty} \frac{k(n)}{n} = \infty.$$

Problem

1. Find more precise estimates of $k(n)$.
2. For $n \geq 4$ is the result from the theorem valid if $f \in C^{j(n)}([0,1])$ with $j(n) < k(n)$?

Dorian Popa, Ioan Raşa

4. Problem.

The problem concerns the stability behaviour of the functional equation

$$F(y) - F(x) = (y-x)[a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)] \tag{1}$$

which stems from quadrature rules of numerical integration. Examples of particular cases of (1) are given by

$$F(y) - F(x) = (y-x) f\Big(\frac{x+y}{2}\Big), \tag{2}$$

$$F(y) - F(x) = (y-x)[f(x) + f(y)], \tag{3}$$

$$F(y) - F(x) = (y-x)\Big[\frac{1}{6} f(x) + \frac{2}{3} f\Big(\frac{x+y}{2}\Big) + \frac{1}{6} f(y)\Big] \tag{4}$$

and many others, see for example [1].

Thus we deal with the following inequality

$$|F(y) - F(x) - (y-x)[a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)]| \leq \varepsilon. \tag{5}$$

Substituting first $x+h$ in place y, then $x+2h, x+h$ instead of y, x resp. and, finally, $x+2h$ instead of y it is possible to eliminate F from (5).

What we get is an equation of the shape

$$|h[b_1 f(\gamma_1 x + \delta_1 h) + \ldots + b_n f(\gamma_{2n+1} x + \delta_{2n+1} h)]| \leq 3\varepsilon$$

for some b_i, γ_i, δ_i depending on a_i, α_i, β_i occurring in (5).

For example if we start with the inequality

$$\Big|F(y) - F(x) - f\Big(\frac{x+y}{2}\Big)\Big| \leq \varepsilon,$$

then we get

$$|h\Delta_h^2 f(x)| \leq 3\varepsilon.$$

In this case it is possible to prove that f satisfies

$$\Delta_h^2 f(x) = 0$$

(see [2]), which yields the superstability of (2). The same procedure may be applied to (3).

However the stability behaviour of (even slightly) more complicated equations stemming from numerical analysis such as (for example) (4) is unknown. It is easy to see that, using our procedure with respect to

$$\left|F(y) - F(x) - (y-x)\Big[\frac{1}{6}f(x) + \frac{2}{3}f\Big(\frac{x+y}{2}\Big) + \frac{1}{6}f(y)\Big]\right| \le \varepsilon,$$

we obtain

$$|h\Delta_h^4 f(x)| \le 3\varepsilon$$

but is impossible to repeat the idea used in [2] to show that every function satisfying this inequality must be a true solution of $\Delta_h^4 f(x) = 0$. Therefore a natural question arises if equations of the type

$$h\Delta_h^n f(x) = 0$$

are stable or not.

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Tomasz Szostok

List of Participants

1. **ABDULHADI Zayid**, American University of Sharjah, 26666 Sharjah, United Arab Emirates, email: zahadi@aus.edu
2. **ADAM Marcin**, Institute of Mathematics, Silesian University of Technology, Kaszubska 23, 44-100 Gliwice, Poland, email: marcin.adam@polsl.pl
3. **ALESTALO Pekka**, Department of Mathematics and Systems Analysis, School of Science, Aalto University, Otakaari 1 F, 11100, 76 Aalto, Espoo, Finland, email: pekka.alestalo@aalto.fi
4. **ANDRÁS Szilárd**, Mathematics and Computer Science, Babeş-Bolyai University, Eroilor 246, Bl. 4D, Ap. 2, 407280 Floresti, Romania, email: andrasz@math.ubbcluj.ro
5. **BADORA Roman**, Institute of Mathematics, University of Silesia, Bankowa 14, 40-007 Katowice, Poland, email: robadora@ux2.math.us.edu.pl
6. **BAHYRYCZ Anna**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: bah@up.krakow.pl
7. **BATKO Bogdan**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: bbatko@up.krakow.pl
8. **BOROS Zoltán**, Institute of Mathematics, University of Debrecen, Pf. 12, 4010 Debrecen, Hungary, email: zboros@science.unideb.hu
9. **BRZDĘK Janusz**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: jbrzdek@up.krakow.pl

10. **CĂDARIU-BRĂILOIU Liviu**, Department of Mathematics, Politehnica University of Timişoara, Piata Victoriei, No.2, 300006 Timişoara, Romania, email: liviu.cadariu-brailoiu@upt.ro
11. **CHENG Lixin**, School of Mathematical Sciences, Xiamen University, Sing-Ming-Nan Rd 422, 361005 Xiamen, China, email: lxcheng@xmu.edu.cn
12. **CHMIELIŃSKI Jacek**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: jacek@up.krakow.pl
13. **CHUDZIAK Jacek**, Faculty of Mathematics and Natural Sciences, University of Rzeszów, Pigonia 1, 35-959 Rzeszów, Poland, email: chudziak@univ.rzeszow.pl
14. **CIEPLIŃSKI Krzysztof**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: kc@up.krakow.pl
15. **CZERNI Marek**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: mczerni@ap.krakow.pl
16. **DVOŘÁKOVÁ Jana**, Mathematical Institute, Silesian University in Opava, Na Rybnicku 626/1, 746 01 Opava, Czech Republic, email: jana.dvorakova@math.slu.cz
17. **EGHBALI Nasrin**, Faculty of Mathematics and Applications, University of Mohaghegh Ardabili, Daneshgah Street, 179 Ardebil, Iran, email: nasrineghbali@gmail.com; eghbali@uma.ac.ir
18. **FÖRG-ROB Wolfgang**, Institute of Mathematics, University of Innsbruck, Technikerstrasse 19a, 6020 Innsbruck, Austria, email: wolfgang.foerg-rob@uibk.ac.at
19. **FOŠNER Ajda**, Faculty of Management, University of Primorska, Cankarjeva 5, 6104 Koper, Slovenia, email: ajda.fosner@uni-mb.si
20. **GILÁNYI Attila**, Faculty of Informatics, University of Debrecen, Egyetem ter 1, 4032 Debrecen, Hungary, email: gilanyi@inf.unideb.hu
21. **GOLEŢ Ioan**, Department of Mathematics, Politehnica University of Timişoara, Piata Victoriei No.2, 300006 Timişoara, Romania, email: ioan.golet@upt.ro
22. **GSELMANN Eszter**, Institute of Mathematics, University of Debrecen, Egyetem ter 1, 4032 Debrecen, Hungary, email: gselmann@science.unideb.hu
23. **JABŁOŃSKA Eliza**, Departament of Mathematics, Rzeszów University of Technology, Powstańców Warszawy 12, 35-959 Rzeszów, Poland, email: elizapie@prz.edu.pl
24. **JABŁOŃSKI Wojciech**, Department of Mathematics, Rzeszów University of Technology, Powstańców Warszawy 12, 35-959 Rzeszów, Poland, email: wojtek@prz.edu.pl
25. **KOČAN Zdeněk**, Mathematical Institute, Silesian University in Opava, Na Rybnicku 626/1, 746 01 Opava, Czech Republic, email: zdenek.kocan@math.slu.cz
26. **KOZYRA Magdalena**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: magdalena.kozyra1987@gmail.com
27. **KUREK Paulina**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: paulinakurek11@gmail.com
28. **LEŚNIAK Zbigniew**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: zlesniak@up.krakow.pl
29. **MALEJKI Renata**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: smalejki@o2.pl
30. **MÉSZÁROS Alpár Richárd**, Laboratoire de Methématique d'Orsay, Mathematics, Univerité Paris-Sud, Bat. 425, 91495 Orsay, France, email:alpart_r@yahoo.com
31. **NAGY Tímea**, CMAP, École Polytechnique, Palaiseau, France, email: timea.nagy@polytechnique.edu
32. **NIKODEM Kazimierz**, Department of Mathematics and Computer Science, University of Bielsko-Biala, Willowa 2, 43-309 Bielsko-Biała, Poland, email: knikodem@ath.bielsko.pl
33. **OLBRYŚ Andrzej**, Institute of Mathematics, University of Silesia, Bankowa 14, 40-007 Katowice, Poland, email: andrzej.olbrys@wp.pl

34. **OLKO Jolanta**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: jolko@up.krakow.pl
35. **OUBBI Lahbib**, Ecole Normale Supérieure, Mohammed V-Agdal, Avenue Mohamed Hassan El Ouazzan, 10105 Rabat, Morocco, email: oubbi@daad-alumni.de
36. **PÁLES Zsolt**, Institute of Mathematics, University of Debrecen, Egyetem ter 1, 4032 Debrecen, Hungary, email: pales@science.unideb.hu
37. **PASTECZKA Paweł**, Institute of Mathematics, University of Warsaw, Krakowskie Przedmieście 26/28, 00-927 Warszawa, Poland, email: ppasteczka@mimuw.edu.pl
38. **PISZCZEK Magdalena**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: magdap@ap.krakow.pl
39. **POPA Dorian**, Automation and Computer Science, Technical University of Cluj-Napoca, C. Daicoviciu 28, 400114 Cluj-Napoca, Romania, email: Popa.Dorian@math.utcluj.ro
40. **PRZEBIERACZ Barbara**, Institute of Mathematics, University of Silesia, Bankowa 14, 40-007 Katowice, Poland, email: barbara.przebieracz@us.edu.pl
41. **RAŞA Ioan**, Automation and Computer Science, Technical University of Cluj-Napoca, C. Daicoviciu 28, 400114 Cluj-Napoca, Romania, email: Ioan.Rasa@math.utcluj.ro
42. **ŞEVGIN Sebaheddin**, Department of Mathematics, Yüzüncü Yil University, 65080 Van, Turkey, email: ssevgin@yahoo.com
43. **ŞEVLI Hamdullah**, Department of Mathematics, Faculty of Science, Istanbul Commerce University, Sütlüce / Beyoglu, Istanbul, Turkey, email: hsevli@yahoo.com
44. **SIKORSKA Justyna**, Institute of Mathematics, University of Silesia, Bankowa 14, 40-007 Katowice, Poland, email: sikorska@math.us.edu.pl
45. **SIUDUT Stanisław**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: siudut@ap.krakow.pl
46. **ŚLIWIŃSKA Dorota**, Department of Mathematics and Computer Science, University of Bielsko-Biala, Willowa 2, 43-309 Bielsko-Biała, Poland, email: dsliwinska@ath.bielsko.pl
47. **SMÍTAL Jaroslav**, Mathematical Institute, Silesian University in Opava, Na Rybnicku 626/1, 746 01 Opava, Czech Republic, email: Jaroslav.Smital@math.slu.cz
48. **SOBEK Barbara**, Faculty of Mathematics and Natural Sciences, University of Rzeszów, Pigonia 1, 35-310 Rzeszów, Poland, email: b_sobek@wp.pl
49. **SOLARZ Paweł**, Departament of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: psolarz@up.krakow.pl
50. **ŠTEFÁNKOVÁ Marta**, Mathematical Institute, Silesian University in Opava, Na Rybnicku 626/1, 746 01 Opava, Czech Republic, email: marta.stefankova@math.slu.cz
51. **STEVIĆ Stevo**, Mathematical Institute of the Serbian Academy of Sciences, Knez Mihailova 36/III, 11000 Beograd, Serbia, email: sstevic@ptt.rs
52. **SZAŁA Leszek**, Mathematical Institute, Silesian University in Opava, Na Rybnicku 626/1, 74601 Opava, Czech Republic, email: leszek.szala@math.slu.cz
53. **SZCZAWIŃSKA Joanna**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: jszczaw@ap.krakow.pl
54. **SZÉKELYHIDI László**, Institute of Mathematics, University of Debrecen, Petöfi Sandor, 9222 Hegyeshalom, Hungary, email: lszekelyhidi@gmail.com
55. **SZOSTOK Tomasz**, Institute of Mathematics, University of Silesia, Bankowa 14, 40-007 Katowice, Poland, email: szostok@math.us.edu.pl
56. **TABOR Jacek**, Faculty of Mathematics and Computer Science, Jagiellonian University, Łojasiewicza 6, 30-348 Kraków, Poland, email: tabor@ii.uj.edu.pl
57. **VOLKMANN Peter**, Institut für Analysis, KIT, 76128 Karlsruhe, Germany
58. **WĄSOWICZ Szymon**, Department of Mathematics and Computer Science, University of Bielsko-Biala, Willowa 2, 43-309 Bielsko-Biała, Poland, email: swasowicz@ath.bielsko.pl

59. **WÓJCIK Paweł**, Department of Mathematics, Pedagogical University, Podchorążych 2, 30-084 Kraków, Poland, email: pwojcik@up.krakow.pl

60. **ZLATOŠ Pavol**, Department of Algebra, Geometry and Mathematics Education, Comenius University, Mlynska dolina, 84248 Bratislava, Slovakia, email: Pavol.Zlatos@fmph.uniba.sk

(Compiled by Jolanta Olko and Magdalena Piszczek)