FOLIA 128

Annales Universitatis Paedagogicae Cracoviensis

Studia Mathematica XII (2013)

Report of Meeting

15th International Conference on Functional Equations and Inequalities, Ustroń, May 19-25, 2013

Contents

The Fifteenth International Conference on Functional Equations and Inequalities was held from May 19 to 25, 2013 in Ustroń, Poland. The series of ICFEI meetings has been organized by the Department of Mathematics of the Pedagogical University in Cracow since 1984. As usual, the ICFEI meeting was focused on various topics connected with functional equations and inequalities as well as on their applications.

The Organizing Committee of the 15th ICFEI consisted of Janusz Brzdęk as Chairman, Krzysztof Ciepliński and Zbigniew Leśniak as Vice-Chairmans, Anna Bahyrycz and Magdalena Piszczek as Scientific Secretaries, Paweł Solarz, Janina Wiercioch and Paweł Wójcik.

The Scientific Committee consisted of Professors: Dobiesław Brydak as Honorary Chairman, Janusz Brzdęk as Chairman, Nicole Brillouët-Belluot, Jacek Chmieliński, Roman Ger, Hans-Heinrich Kairies, László Losonczi, Zsolt Páles, Ekaterina Shulman, László Szekelyhidi and Marek Cezary Zdun.

The 82 participants came from 21 countries: Armenia, Austria, Brasil, Canada, China, Croatia, Czech Republic, Denmark, France, Germany, Hungary, Israel, Japan, Luxemburg, Romania, Russia, Serbia, Slovenia, USA, Venezuela and Poland. The 23 of them were the first-time ICFEI attendees.

The conference was opened on Monday, May 20 by Professor Janusz Brzdęk – the Chairman of the Scientific and Organizing Committees, who welcomed the participants on behalf of the Organizing Committee (and read a letter to them from Professor Władysław Błasiak, the Dean of the Faculty of Mathematics, Physics and Technical Science of the Pedagogical University). The opening address was given by Professor Jacek Chmieliński, the Head of the Department of Mathematics of the Pedagogical University.

Altogether, during 21 scientific sessions 69 talks were given. They focused on functional equations in a single variable and in several variables, functional inequalities, stability theory, convexity, multifunctions, means and other topics. Several contributions have been made during special Problems and Remarks sessions.

On Tuesday, May 20, a picnic was organized. On the next day afternoon the participants visited Cieszyn, a town situated in the heart of the historical region of Cieszyn Silesia, on the border with the Czech Republic. According to the legend, Cieszyn was established in 810 and up to the present, the town has preserved its medieval urban plan. On Thursday, May 22, a banquet was held. The conference was closed on Saturday, May 25 by Professor Janusz Brzdęk. He announced that the 16th ICFEI will be organized in Mathematical Research and Conference Center in Będlewo.

The following part of the report contains the abstracts of the talks, the problems and remarks, and a list of the participants with their addresses.

Abstracts of Talks

Shoshana Abramovich *Refined Jensen's inequalities and Hardy's inequalities*

We present a new set of functions and show how inequalities satisfied by functions of this set lead to and refine Jensen's type inequalities, Hardy's type inequalities and many other inequalities.

For Hardy's inequality the "breaking point", that is the point where the inequality reverses, is $p = 1$. Here we prove that for a function in our set the refined Hardy's type inequality can have a breaking point at any $p \geq 1$.

Marcin Adam *On the difference property of higher orders for differentiable functions*

Let $C^p(\mathbb{R},\mathbb{R})$ denote the class of p-times continuously differentiable functions. Inspired by some results concerning the double difference property [1], we show that the class C^p has the difference property of p-th order, i.e. if a function $f\colon \mathbb{R} \to \mathbb{R}$ is such that $\Delta_h^p f \in C^p(\mathbb{R}\times\mathbb{R},\mathbb{R})$, where $\Delta_h^p f$ is the p-th iterative of the well-known difference operator $\Delta_h f(x) := f(x+h) - f(x)$, then there exists a unique polynomial function $P\colon \mathbb{R} \to \mathbb{R}$ of $(p-1)$-th order such that $f - P \in C^p(\mathbb{R},\mathbb{R})$. Moreover, the function P is given by the formula

$$P(x) = f(x) - \frac{1}{p!}\int_0^1\int_0^{t_1}\int_0^{t_2}\dots\int_0^{t_{p-1}} \partial_2^p(\Delta^p f)(ux,0)(x^p)\,du\,dt_{p-1}\dots dt_1, \qquad x \in \mathbb{R}.$$

Some new equalities connected with the difference operator are also presented.

[1] J. Tabor, J. Tabor, *Stability of the Cauchy type equations in the class of differentiable functions*, J. Approx. Theory **98** (1999), 167–182.

Roman Badora *Remarks on Hyers theorem*
(joint work with **B. Przebieracz** and **P. Volkmann**)

Let Y be a linear space and let B be a subset of Y. We are looking for a condition on the set B which guarantees that for every commutative semigroup S and each function $f: S \to Y$ fulfilling

$$f(s+t) - f(s) - f(t) \in B, \qquad s, t \in S$$

there exists an additive function $a: S \to Y$ such that

$$a(s) - f(s) \in B, \qquad s \in S.$$

Anna Bahyrycz *Approximately p-Wright affine functions and inner product spaces*
(joint work with **J. Brzdęk** and **M. Piszczek**)

We consider the equation of p-Wright affine functions

$$g(px + (1-p)y) + g((1-p)x + py) = g(x) + g(y) \tag{1}$$

with a fixed $p \in \mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$, for functions g mapping a normed space over $\mathbb{F}$ into a normed space.

We present a result on hyperstability of equation (1) and show that for

$$p = \frac{e^{i\alpha} + 1}{2}$$

with some $\alpha \in \mathbb{R}$, equation (1) characterizes norms in the complex inner product spaces. We also obtain in this way some inequalities describing derivations, Lie derivations and Lie homomorphisms.

Karol Baron *On additive involutions and Hamel bases*

Let X be a linear topological space, $X \neq \{0\}$, put

$$\mathcal{A} = \{a: X \to X \mid a \text{ is additive}\}$$

and consider $\mathcal{A}$ with the topology induced by X^X with the Tychonoff topology. Inspired by the foot-note on p.325 of [2] (on p.294 of the original edition) and making use of the lemma from [1] we show that the following sets are dense in $\mathcal{A}$:

$$\{a \in \mathcal{A}: \ a \circ a = \mathrm{id}_X,\ a \text{ is discontinuous and } a(H) \setminus H \neq \emptyset \text{ for every}$$
$$\text{uncountable set } H \subset X \text{ which is linearly independent over } \mathbb{Q}\},$$

$$\{a \in \mathcal{A}: \ a \circ a = \mathrm{id}_X,\ a \text{ is discontinuous and } a(H) = H \text{ for a basis } H$$
$$\text{of the vectorspace } X \text{ over the field } \mathbb{Q}\}.$$

[1] K. Baron, P. Volkmann, *Dense sets of additive functions*, Seminar LV, No. 16 (2003), 4 pp., http://www.math.us.edu.pl/smdk.

[2] M. Kuczma, *An introduction to the theory of functional equations and inequalities. Cauchy's equation and Jensen's inequality*, Second edition (edited by A. Gilányi), Birkhäuser Verlag, Basel (2009).

Bogdan Batko *Stability of the exponential Cauchy functional equation in Riesz algebras*

We deal with the stability of the exponential Cauchy functional equation

$$f(x+y) = f(x)f(y) \qquad \text{for } x, y \in G$$

in the class of functions $f\colon G \to L$ mapping a group $(G,+)$ into a Riesz algebra L.

Janusz Brzdęk *Stability of the Cauchy equation – revisited*

The following theorem is considered to be one of the most classical results concerning stability of the additive Cauchy equation.

Theorem 1
Let E_1 and E_2 be normed spaces, E_2 be complete, $c > 0$ and $p \neq 1$ be fixed real numbers, and $f\colon E_1 \to E_2$ satisfy

$$\|f(x+y) - f(x) - f(y)\| \le c\,(\|x\|^p + \|y\|^p), \qquad x, y \in E_1 \setminus \{0\}. \tag{1}$$

Then there exists a unique additive function $T\colon E_1 \to E_2$ with

$$\|f(x) - T(x)\| \le \frac{c\|x\|^p}{|2^{p-1} - 1|}, \qquad x \in E_1 \setminus \{0\}.$$

The particular cases of it have been proved by D.H. Hyers, T. Aoki, Th.M. Rassias and Z. Gajda. We present some comments and recent results connected with it. In particular, the following holds true.

Theorem 2
Let E_1 and E_2 be normed spaces, $c > 0$ and $p < 0$ be fixed real numbers, and $f\colon E_1 \to E_2$ satisfy (1). *Then f is additive.*

Liviu Cădariu-Brăiloiu *Fixed points and generalized stability of some functional equations*
(joint work with **I. Goleţ**)

The fixed point method is an extensive technique used for proving the Hyers-Ulam stability of functional equations. The goal of this talk is to present applications of some fixed point theorems to the theory of Hyers-Ulam stability of several functional equations.

[1] J. Brzdęk, J. Chudziak, Z. Páles, *A fixed point approach to stability of functional equations*, Nonlinear Anal. **74** (2011), 6728–6732.

[2] L. Cădariu, V. Radu, *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure Appl. Math. **4** (2003), Article 4, 7 pp. (electronic).

[3] L. Cădariu, V. Radu, *Fixed point methods for the generalized stability of functional equations in a single variable*, Fixed Point Theory Appl. (2008), Art. ID 749392, 15 pp.

[4] L. Cădariu, L. Găvruţa, P. Găvruţa, *Weighted space method for the stability of some nonlinear equations*, Appl. Anal. Discrete Math. **6** (2012), 126–139.

[5] L. Cădariu, L. Găvruţa, P. Găvruţa, *Fixed points and generalized Hyers-Ulam stability*, Abstr. Appl. Anal. (2012), Art.ID 712743, 10 pp.

Jacek Chmieliński *Orthogonality equation with two unknown functions*

We consider generalized orthogonality equations involving two unknown functions:

$$\langle f(x)|g(y)\rangle = \langle x|y\rangle, \qquad \langle f(x)|g(y)\rangle = \langle y|x\rangle, \qquad |\langle f(x)|g(y)\rangle| = |\langle x|y\rangle|.$$

As a related problem, we investigate the orthogonality preserving property

$$x\perp y \implies f(x)\perp g(y).$$

Jacek Chudziak *On solutions of a generalization of the Gołąb-Schinzel functional equation*

Assume that X is a real linear space and let $\phi, \psi\colon \mathbb{R} \to \mathbb{R}$ be continuous functions. We consider solutions of the functional equation

$$f(x\phi(f(y)) + y\psi(f(x))) = f(x)f(y) \qquad \text{for } x, y \in X, \tag{1}$$

where $f\colon X \to \mathbb{R}$ is an unknown function. Equation (1) is a generalization of the Gołąb-Schinzel functional equation

$$f(x + yf(x)) = f(x)f(y)$$

as well as its further generalizations

$$f(xf(y)^k + yf(x)^l) = f(x)f(y),$$

where k and l are fixed positive integers. A particular case of (1), namely the equation

$$f(x(pf(y) + (1-p)) + y((1-p)f(x) + p)) = f(x)f(y),$$

where p is a fixed real number, has been considered in [3]. Continuous solutions of (1) in the case $X = \mathbb{R}$ have been determined in [1] and [2].

[1] J. Chudziak, *Continuous solutions of a generalization of the Gołąb-Schinzel equation*, Aequationes Math. **61** (2001), 63–78.

[2] J. Chudziak, *Continuous solutions of a generalization of the Gołąb-Schinzel equation II*, Aequationes Math. **71** (2006), 115–123.

[3] J. Matkowski, *A generalization of the Gołąb-Schinzel functional equation*, Aequationes Math. **80** (2010), 181–192.

Marek Czerni *Representation theorems for regular solutions of linear functional inequality*

In the talk we present representation theorems for regular solutions of the linear functional inequality

$$\psi[f(x)] \leq g(x)\psi(x) + h(x), \tag{1}$$

where ψ is an unknown function and f, g, h are given.

We assume the following hypotheses about given functions f, g and h:

(H_1) The function $f\colon I \to \mathbb{R}$ is continuous and strictly increasing in an interval $I = [0, a)$, where $0 < a \leq \infty$. Moreover, $0 < f(x) < x$ for $x \in I^\star = I \setminus \{0\}$.

(H_2) The function $g\colon I \to \mathbb{R}$ is continuous in I and $g(x) > 0$ for $x \in I^\star$. Moreover, functional sequence $G_n(x) = \prod_{i=0}^{n-1} g[f^i(x)]$ tends to zero almost uniformly in $I^\star$.

(H_3) The function $h\colon I \to \mathbb{R}$ is continuous in I and functional sequence $\varphi_n^\star(x) = \sum_{i=0}^{n-1} \frac{h[f^i(x)]}{G_{i+1}(x)}$ converges almost uniformly in $I^\star$.

We shall be concerned with such regular solutions of (1) that for some fixed continuous solution φ of the linear functional equation

$$\varphi[f(x)] = g(x)\varphi(x) + h(x)$$

or

$$\varphi[f(x)] = g(x)\varphi(x)$$

the following asymptotic condition

$$\psi[f(x)] = g(x)\psi(x) + h(x) + O(\varphi(x)), \qquad x \to 0^+$$

holds.

The result presented are related to those from [1] (see in particular section 12.5 pp.488–490)

[1] M. Kuczma, B. Choczewski, R. Ger, *Iterative functional equations*, Encyclopedia of Mathematics and its Applications, **32**, Cambridge University Press, Cambridge, 1990.

Thomas M.K. Davison *Near-homomorphisms on semigroups*

If a complex-valued function f defined on a group satisfies d'Alembert's equation

$$f(xy) + f(xy^{-1}) = 2f(x)f(y),$$

then it also satisfies the centrality equation

$$f(xy) = f(yx) \tag{1}$$

and

$$f(xyz) + f(xzy) = 2f(x)f(yz) + 2f(y)f(xz) + 2f(z)f(xy) - 4f(x)f(y)f(z). \tag{2}$$

Now looking at (1) and (2) we see that every semigroup homomorphism satisfies them. This leads to the main concept of this paper.

Definition

Let S be a semigroup and $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ complex numbers, not all zero. Then $f\colon S \to \mathbb{C}$ is a near-homomorphism if f is central (satisfies (1)), and for all x, y, z in S,

$$\begin{aligned}\alpha_1 f(xyz) + \alpha_2 f(xzy) = \beta_1 f(x)f(yz) + \beta_2 f(y)f(xz) + \beta_3 f(z)f(xy)\\ + (\alpha_1 + \alpha_2 - \beta_1 - \beta_2 - \beta_3)f(x)f(y)f(z).\end{aligned}$$

Here is our main result:

Theorem

If f is a near-homomorphism on S, then at least one of the following is true

(i)
$$f(xyz) = f(xzy)$$
for all x, y, z in S.

(ii) *There is a complex number λ such that for all x, y in S,*
$$f(xy) = \lambda f(x)f(y).$$

(iii) *Either f or $\frac{1}{2}f$ satisfies (2).*

Joachim Domsta *Unitary operators with gaussian kernels*

All unitary one-parameter continuous groups on $L^2(\mathbb{R})$ formed by integral operators with kernels of the following gaussian form

$$\mathcal{U}^{(t)}(x,y) = \sqrt{\frac{B_t}{2\pi i}}\exp\Big\{-\frac{1}{2i}\big[A_t x^2 - 2B_t xy + C_t y^2\big]\Big\}, \qquad x, y \in \mathbb{R},\ t \in \mathbb{R},$$

are presented unless the coresponting operator U_t equals the identity.

The following three cases are possible, only (not counting the trivial group of identities)

$$(1) \qquad B_t = \frac{S}{t}, \quad A_t = B_t + R, \quad C_t = B_t - R; \qquad t \neq 0;$$

$$(2) \quad B_t = \frac{S\alpha}{\sinh(\alpha t)}, \quad A_t = B_t\cosh(\alpha t) + R, \quad C_t = B_t\cosh(\alpha t) - R; \qquad t \neq 0;$$

$$(3) \quad B_t = \frac{S\alpha}{\sin(\alpha t)}, \quad A_t = B_t\cos(\alpha t) + R, \quad C_t = B_t\cos(\alpha t) - R; \qquad \alpha t \neq k\pi;$$

for some real R, $S \neq 0$, $\alpha \neq 0$. Correspondingly, the unitary groups cover all quantum dynamical systems on $\mathbb{R}$ driven by the hamiltonians

$$H = -a\frac{\partial^2}{\partial x^2} + bx^2 + ic\Big(x\frac{\partial}{\partial x} + \frac{\partial}{\partial x}x\Big)$$

with real a, b, c. The case of $R = 0$ (equivalently, $c = 0$) covers the standard quantum dynamics of one dimensional free particle (if $b = 0$), possibly under the influence of the quadratic potential (attracting or repulsing "harmonic" oscilator - when $a \cdot b \neq 0$).

El-Sayed El-Hady *On a two-variable functional equation*
(joint work with **W. Förg-Rob** and **H. Nassar**)

Functional equations have applications in many fields such as communications, economics, and information theory. We study a two-variable functional equation which naturally arises from modeling two-queue queueing systems. This functional equation could be solved by reduction to a boundary value problem, most notably to a Riemann-Hilbert boundary value problem. However, the exact form of the solutions for this equation is rarely obtained. We manage to solve a challenged two-variable functional equation arising from a gateway queueing model by trial and error. There we got many solutions which do not make sense with the system under consideration of the application.

Włodzimierz Fechner *Functional inequalities connected with averaging operators*

In 1934 J. Kampé de Fériet [7] introduced the notion of averaging operators. If $\mathcal{A}$ is a function algebra then a linear operator $T\colon \mathcal{A} \to \mathcal{A}$ is called averaging if it satisfies the equality

$$T(f \cdot Tg) = Tf \cdot Tg \tag{1}$$

for each $f, g \in \mathcal{A}$. This idea was developed further by G. Birkhoff [1]. After the Birkhoff's paper was published, this topic was extensively discussed by a number of authors.

Assume that $(G, *)$ is an arbitrary semigroup and $f\colon G \to G$. The following functional equation

$$f(x * f(y)) = f(x) * f(y)$$

is motivated by relation (1) and was studied mainly by J. Dhombres [2]–[6].

In the talk we will investigate special cases of the following functional inequality

$$f(x * f(y)) \geq f(x) * f(y)$$

for the unknown mapping f acting on a partially ordered ring $\mathcal{R}$. We will exhibit two cases, namely when the operation $*$ is equal to addition or multiplication on $\mathcal{R}$, respectively.

[1] G. Birkhoff, *Moyennes des fonctions bornées*, Colloques internationaux du C. N. R. S.: Algèbre et Théorie des nombres **24** (1950), 143–153.

[2] J. Dhombres, *Caractérisation d'une classe de transformations semi-multiplicatives*, C. R. Acad. Sci. Paris Sér. A-B **264** (1967), A113–A116.

[3] J. Dhombres, *A functional characterization of Markovian linear exaves*, Bull. Amer. Math. Soc. **81** (1975), 703–706.

[4] J. Dhombres, *Functional equations, averaging operators, interpolation operators, and linear extension operators* (in Chinese), Nanta Math. **9** (1976), 109–116.

[5] J. Dhombres, *Solution générale sur un groupe abélien de l'équation fonctionnelle* $f(x*f(y)) = f(f(x)*y)$, Aequationes Math. **15** (1977), 173–193.

[6] J. Dhombres, *Some aspects of functional equations*, Chulalongkorn University, Department of Mathematics, Bangkok, 1979.

[7] J. Kampé de Fériet, *L'etat actuel du problème de la turbulence (I and II)*, La Science Aérienne **3** (1934), 9–34; **4** (1935), 12–52.

Carlos E. Finol *An inequality related to geometrically concave functions*

We shall consider the class of functions $f\colon [1,\infty) \to [1,\infty)$, which are twice continuously differentiable in $(1,\infty)$ and such that:

(A1) The function $h(t) := \frac{tf'(t)}{f(t)}$, $t > 1$, is strictly decreasing, $0 < h(t) < 1$ for $t > 1$ and $\lim_{t\to\infty} h(t) = 0$.

(A2) $f(1) = 1$ and $\lim_{t\to\infty} f(t) = \infty$.

(A3) The function $t \mapsto \frac{t}{f(t)}$ is concave.

Lemma
The functions in this class are strictly submultiplicative; that is, for $x, y \in (1,\infty)$, $x \neq y$, we have $f(xy) < f(x)f(y)$.

Let X be a Banach space with unit Schauder basis $\{e_n\}_{n=1}^{\infty}$, and let $\|\cdot\|$ be its norm. An important numerical parameter associated to the unit basis is defined as follows, $\lambda(n) = \|e_1 + e_2 + \ldots + e_n\|$, $n \in \mathbb{N}$.

Theorem
For each $m \in \mathbb{N}$ we have that $\lambda(m) = \frac{m}{f(m)}$, where f satisfies (A1), (A2) *and* (A3).

Ajda Fošner *Some results on the Hyers–Ulam–Rassias stability of functional equations*

A classical question in the theory of functional equations is: *Under what conditions is it true that a mapping which approximately satisfies a functional equation $\mathcal{E}$ must be somehow close to an exact solution of $\mathcal{E}$?* We say that a functional equation $\mathcal{E}$ is stable if any approximate solution of $\mathcal{E}$ is near to a true solution of $\mathcal{E}$. We will present some new results on the generalized Hyers–Ulam–Rassias stability of functional equations.

Ji Gao *Some equations and inequalities related to geometric properties in Banach spaces*
(joint work with **S. Saejung** and **J. Gao**)

A bounded, convex subset K of a Banach space X is said to have normal structure if every convex subset H of K that contains more than one point contains a point $x_0 \in H$, such that $\sup\{\|x_0 - y\| : y \in H\} < \mathrm{diam}(H)$, where $\mathrm{diam}(H) =$

$\sup\{\|x-y\| : x,y \in H\}$ denotes the diameter of H. Normal structure implies fixed point property for non-expansive mapping.

At this talk, we show some equations and inequalities and demonstrate the relationships among parameters $\delta(\epsilon)$, $C(X)$, $\rho_X(\tau)$, $J(X)$, $O(X)$, $Q(X)$ and $\omega(X)$ of X, that imply normal structure. Many results in this field are either improved under a certain condition or obtained in a different way.

Roman Ger *On a problem of Nicolae Bourbăcuţ*

In the April 2012 issue of *The American Mathematical Monthly* (**119**, Problems and Solutions, p.345) the following problem was proposed by Nicolae Bourbăcuţ (Sarmizegetusa, Romania):

Let f be a convex function from $\mathbb{R}$ into $\mathbb{R}$ and suppose that

$$f(x+y)+f(x-y)-2f(x) \le y^2$$

for all real x and y.

(a) Show that f is differentiable.

(b) Show that for all real x and y,

$$|f'(x)-f'(y)| \le |x-y|.$$

(Problem **11641**).

We shall present a solution of the problem in question (in much more general setting) from which the solution of Bourbăcuţ's problem will result as an obvious corollary.

Dorota Głazowska *Invariance equation involving quasi-arithmetic mean and Matkowski means*

Let $I \subset \mathbb{R}$ be an open interval. We consider the following functional equation

$$\varphi\big((f+g)^{-1}(f(x)+g(y))\big)+\varphi\big((f+g)^{-1}(g(x)+f(y))\big) = \varphi(x)+\varphi(y), \qquad x,y \in I,$$

where $\varphi, f, g\colon I \to \mathbb{R}$ are continuous and strictly monotone functions such that f and g are strictly monotone in the same sense. We solve this equation under two times continuous differentiability of the unknown functions φ, f, g.

Attila Házy *On approximately (k,h)-convex functions*

In our talk we define the so-called approximately (k,h)-convex function with respect to a set, which is a natural generalization of the usual convexity, the s-convexity in the first (Orlicz) and second (Breckner) sense, the h-convexity, the Godunova-Levin functions and the P-functions. We investigate some regularity and Bernstein-Doetsch type results for this type functions.

Let X be a real or complex topological vector space, T be a nonempty set such that the following property holds

$$t \in T \text{ if and only if } 1-t \in T.$$

Furthermore, let $k, h\colon T \to \mathbb{R}$ be given functions and $D \subset X$ be a nonempty open, k-convex set (that is, $k(t)x + k(1-t)y \in D$ whenever $x, y \in D$ and $t \in T$). We say that a function $f\colon D \to \mathbb{R}$ is approximately (k,h)-convex with respect to T, if

$$f(k(t)x + k(1-t)y) \le h(t)f(x) + h(1-t)f(y) + d(x,y)$$

for all $x, y \in D$ and $t \in T$, where $d\colon X \times X \to \mathbb{R}$ is a given function.

Dijana Ilišević *Orthogonally additive mappings on Hilbert C^*-modules* (joint work with **A. Turnšek** and **Dilian Yang**)

Let $\mathcal{A}$ be a C^*-algebra and let $(W, \langle\,.\,,.\,\rangle)$ be a Hilbert C^*-module over $\mathcal{A}$. A mapping f on W is said to be orthogonally additive if for all $x, y \in W$

$$\langle x, y\rangle = 0 \implies f(x+y) = f(x) + f(y).$$

If T is an additive mapping on W and Φ is an additive mapping on $\mathcal{A}$, then the mapping f defined by

$$f(x) = T(x) + \Phi(\langle x, x\rangle) \qquad \text{for } x \in W$$

is an orthogonally additive mapping. The aim of this talk is to answer the question whether the converse also holds true.

Hideaki Izumi *Formal series solutions of iterative functional equations*

We will introduce φ-power series

$$f(x) = a_1\varphi(x) + a_2\varphi(x)^2 + \ldots + a_n\varphi(x)^n + \ldots,$$

where $a_n \in \mathbb{R}$, $n = 1, 2, \ldots$ and φ is a given function analytic in the neighborhood of 0 satisfying $\varphi(0) = 0$, $\varphi'(0) = 1$. We will show that for each $n \in \mathbb{N}$ the iterative functional equation

$$f^n(x) = \varphi(x)$$

has a formal solution. Moreover, we will discuss the radius of convergence of the formal solution.

Wojciech Jabłoński *Regular groups of formal power series commuting in pairs*

Let R be an integral domain of characteristic 0. We consider groups $\mathcal{F}^s$ of invertible formal power series $\sum_{k=1}^{s} c_k X^k \in \Gamma^s$ commuting in pairs, i.e. satisfying

$$(F_1 \circ F_2)(X) = (F_2 \circ F_1)(X) \qquad \text{for } F_1(X), F_2(X) \in \mathcal{F}^s.$$

In some cases these groups are the regular one-parameter groups $(F(t,X))_{t\in T}$ of formal power series, which means that the formal derivative $\frac{\partial F}{\partial t}(t,X)$ exists for each $t \in T$. We obtain formal differential equations in the ring of formal power series over the field $\mathbb{K}_R$ of fractions of the ring R.

Justyna Jarczyk *Uniform convexity of paranormed generalizations of L^p spaces* (joint work with **J. Matkowski**)

For a measure space (Ω, Σ, μ) and a bijective increasing function $\varphi\colon [0,\infty) \to [0,\infty)$ the L^p-like paranormed function space with the paranorm of the form

$\mathbf{p}_\varphi(x) = \varphi^{-1}(\int_\Omega \varphi \circ |x|\, d\mu)$ is considered. Main results give general conditions under which this space is uniformly convex. The Clarkson theorem on the uniform convexity of L^p-space is generalized. Under some specific assumptions imposed on φ we give not only theorems on the uniform convexity but also formulas of modulus of convexity.

Witold Jarczyk *Iterative roots of piecewise monotonic functions revisited* (joint work with **J. Jarczyk**, **Lin Li**, **Liu Liu** and **Weinian Zhang**)

We continue research presented in [Liu Liu, W. Jarczyk, Lin Li and Weinian Zhang, Nonlinear Anal. 75 (2012), 286–03], where the so-called case $\mathcal{T}_1$ was studied. The talk reports a progress made recently while developing the complementary case $\mathcal{T}_2$.

Sándor Jenei *On the geometry of associativity*

A two geometric descriptions of associativity of residuated operations will be presented from [3] along with their possible applications for investigating even non-continuous (only left-continuous) solutions of the associativity equation. In particular, we shall show an answer ([2]) to a problem about convex combinations of certain associative functions, posed in [1].

Acknowledgement
Supported by the SROP-4.2.2.C-11/1/KONV-2012-0005 grant and the MC ERG grant 267589.

[1] C. Alsina, M.J. Frank, B. Schweizer, *Problems on associative functions*, Aequationes Math. **66**, (2003), 128–140.

[2] S. Jenei, *On the convex combination of left-continuous t-norms*, Aequationes Math. **72** (2006), 47–59.

[3] S. Jenei, *On the geometry of associativity*, Semigroup Forum **74** (2007), 439–466.

Gergely Kiss *Linear functional equations and derivations* (joint work with **M. Laczkovich**)

We investigate the structure of solutions of the linear functional equation

$$\sum_{i=1}^{n} a_i f(b_i x + c_i y) = 0, \qquad x, y \in \mathbb{C},$$

where a_i, b_i, c_i are given complex numbers, and the numbers b_i/c_i are distict. In order to describe the solutions on the field K generated by the numbers b_i, c_i, we introduce a new method which is related to abstract spectral synthesis on abelian groups. We show that spectral synthesis holds in the variety of the solutions by presenting a dense subset of the variety in terms of the injective homomorphisms and derivations on K.

Zdeněk Kočan *Solutions of a conditional composite type functional equation* (joint work with **J. Chudziak**)

Let X be a real linear space and $C \subset X$ be a convex cone. We deal with the solutions of the following functional equation

$$f(x+g(x)y)=f(x)f(y) \qquad \text{whenever } x,y,x+g(x)y\in C,$$

where $f,g\colon C\to\mathbb{R}$ are unknown functions.

Tomasz Kochanek *Stability of disjointness preserving in C^*-algebras*

We deal with non-commutative analogues of some results, due to Araujo, Font and Dolinar, concerning linear operators defined on $C(X)$-spaces that almost preserve disjointness (that is, the operators $T\colon C(X)\to C(Y)$ such that $f\cdot g=0$ implies $\|T(f)\|\|T(g)\|\leq\varepsilon\|f\|\cdot\|g\|$ for all $f,g\in C(X)$ and for some $\varepsilon\geq 0$). We shall deal with the corresponding property for operators $T\colon\mathcal{A}\to\mathcal{B}$, where $\mathcal{A}$ and $\mathcal{B}$ are C^*-algebras. In particular, we are interested in the question of identifying those properties of the spectra of $\mathcal{A}$ and $\mathcal{B}$ which guarantee the stability effect for operators $\mathcal{A}\to\mathcal{B}$ that almost preserve disjointness.

Bartosz Kołodziejek *The Olkin-Baker functional equation on symmetric cones*

In the talk we will discuss the Olkin-Baker functional equation defined on the cone of symmetric positive definite matrices $\mathcal{V}$ (or generally on symmetric cones)

$$f_1(x)+f_2(y)=f_3(x+y)+f_4(w_i^{-1}(x+y)x), \qquad (x,y)\in\mathcal{V}^2,\ i=1,2,$$

where $w_1(x)y=x^{\frac12}yx^{\frac12}$, $w_2(x)y=t_xyt_x^T$ and t_x is the upper triangular matrix in the Cholesky decomposition of $x\in\mathcal{V}$. It is known that both “multiplication algorithms” w_i give different solutions under assumption of twice differentiability of respective functions. The assumption is now reduced to continuity only.

For the purpose of the proof, the general solutions of the Cauchy functional equation with respect to w_i on positive definite symmetric matrices are found. If the following equation holds for every $(x,y)\in\mathcal{V}^2$

$$g_i(x)+g_i(y)=g_i(w_i(x)y), \qquad i=1,2,$$

then $g_1(x)=H(\det(x))$ and $g_2(x)=\sum_{k=1}^n H_k(\det^{(k)}(x))$, where $\det^{(k)}(x)$ is the kth principal minor of x, n is the rank of $\mathcal{V}$ and $H,H_1,\dots,H_n$ are generalized logarithmic functions.

The Olkin-Baker functional equation is related to the characterization of a Wishart probability distribution on $\mathcal{V}$ (Lukacs-Olkin-Rubin theorem).

Aleksandar Krapež *An application of quasigroup equations in cryptography*

A general solution in closed form for all generalized quadratic quasigroup equations is given. The solution depends on isostrophy classes of quasigroup operations and the tree of the equation. An application of this result in the choice of quasigroups suitable for parastrophic stream cyphers is presented.

Acknowledgement
Supported by the Ministry of Education, Science and Technological Development of Serbia through projects ON 174008 and ON 174026.

Zbigniew Leśniak *On approximate solutions of the generalized Volterra integral equation*
(joint work with **A. Bahyrycz** and **J. Brzdęk**)

We prove some results on approximate solutions of the following generalized Volterra integral equation

$$\psi(x) = \int_a^x N(x, t, \psi(\alpha(t)))\, dt + G(x), \qquad x \in I$$

for continuous functions ψ mapping a real interval I equal to $[a, b)$ or $[a, b]$ into a Banach space B, where $N\colon I \times I \times B \to B$, $G\colon I \to B$, $\alpha\colon I \to I$ are given continuous functions and $\int$ denotes the Bochner integral. We show that, under suitable assumptions, they generate exact solutions of the equation.

Lin Li *On conjugacy of r-modal interval maps with nonmonotonicity height equal to 1*
(joint work with **Z. Leśniak** and **Yong-Guo Shi**)

We construct all homeomorphic solutions and continuously non-monotone solutions of the conjugacy equation $\varphi \circ f = g \circ \varphi$, where $f\colon I \to I$, $g\colon J \to J$ are two given r-modal interval maps with nonmonotonicity height equal to 1, and I, J are closed intervals. Moreover, some sufficient conditions are also presented for the existence of C^1 homeomorphic solutions and C^1 non-monotone solutions, respectively.

Radosław Łukasik *Stability of some generalization of the quadratic and Wilson's functional equation*

In the present talk, we consider the stability of functional equation

$$\sum_{\lambda \in K} f(x + \lambda y) = |K|\alpha(y)g(x) + h(y), \qquad x, y \in S,$$

where $(S, +)$ is an abelian group, K is a finite, abelian subgroup of automorphism group on S, $L := |K|$, $(X, \|\cdot\|)$ is a Banach space over the field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, $f, g, h\colon S \to X$, $\alpha\colon S \to \mathbb{K}$.

We present the stability results in the case of the constant control function.

[1] R. Badora, *On Hyers-Ulam stability of Wilson's functional equation*, Aequationes Math. **60** (2000), 211–218.

[2] R. Badora, *On the stability of a functional equation for generalized trigonometric functions*, Functional equations and inequalities, 1–5, Math. Appl. 518, Kluwer Acad. Publ., Dordrecht, 2000.

[3] R. Badora, *Stability properties of some functional equations*, Functional Equations in Mathematical Analysis, **52** (2012), 3–13.

[4] R. Łukasik, *Some generalization of Cauchy's and quadratic functional equations*, Aequationes Math. **83** (2012), 75–86.

[5] R. Łukasik, *Some generalization of the quadratic and Wilson's functional equation*, Aequationes Math. DOI: 10.1007/s00010-013-0185-y.

Ewelina Mainka-Niemczyk *Sine and cosine families, and series*

Let K be a convex cone in a normed linear space X and let $E_t: K \to n(K)$, $F_t: K \to n(X)$ for $t \geq 0$. A family $\{E_t : t \geq 0\}$ is called a sine family associated with a family $\{F_t : t \geq 0\}$ if

$$E_{t+s}(x) = E_{t-s}(x) + 2F_t(E_t(x)), \qquad 0 \leq s \leq t,\ x \in K,$$

while a family $\{F_t : t \geq 0\}$ is called a cosine family if

$$F_0(x) = \{x\}, \quad F_{t+s}(x) + F_{t-s}(x) = 2F_t(F_s(x)), \qquad 0 \leq s \leq t,\ x \in K$$

(here of course under assumption that values of F_t are in K).

In the talk a necessary and sufficient condition for a family given by some series to be a regular cosine family is presented. Moreover, assumptions under which a regular cosine and sine families can be expressed by series are given.

Judit Makó *On Hermite-Hadamard type inequalities*
(joint work with **A. Házy**)

In this talk, the connection between the functional inequalties

$$f\Big(\frac{x+y}{2}\Big) \leq \frac{f(x)+f(y)}{2} + \alpha_J(x-y), \qquad x, y \in I$$

and

$$\int\limits_{[0,1]} f(tx+(1-t)y)\,d\mu(t) \leq \lambda f(x) + (1-\lambda)f(y) + \alpha_H(x-y), \qquad x, y \in I$$

is investigated, where I is a real interval of $\mathbb{R}$, $f: I \to \mathbb{R}$, $\alpha_H, \alpha_J: \mathbb{R} \to \mathbb{R}$ are even functions, $\lambda \in \mathbb{R}$ and μ is a Borel probability measure on $[0,1]$.

Gyula Maksa *Additive functions which differentiate elementary functions in some sense*

A real derivation is a function $d: \mathbb{R} \to \mathbb{R}$ for which the functional equations

$$d(x+y) = d(x) + d(y), \qquad x, y \in \mathbb{R} \tag{1}$$

and

$$d(xy) = xd(y) + yd(x), \qquad x, y \in \mathbb{R} \tag{2}$$

hold simultaneously. The solutions of (1) are called additive functions and, somewhat surprisingly, it is true that there is a non-zero additive function d that fulfils (2), too. Let $I \subset \mathbb{R}$ be an interval of positive length and $\varphi: I \to \mathbb{R}$ be the differentiable function. We say that the additive function $d: \mathbb{R} \to \mathbb{R}$ differentiates φ if

$$d(\varphi(x)) = \varphi'(x)d(x), \qquad x \in I.$$

In this talk, we present that an additive function which differentiates any of a function from a rather rich list of so-called elementary functions, in the sense above, is real derivation. Furthermore, we discuss the partially exceptional case of power functions and open problems will also be formulated.

Renata Malejki *On stability of Volterra type integral equations in a complex domain*
(joint work with **J. Brzdęk** and **Z. Leśniak**)

We consider Volterra type integral equations of the first and the second order on a simply connected region $D \subset \mathbb{C}$. We prove some Hyers-Ulam stability results using a fixed point theorem for a complete extended metric space of all analytic function defined on D and a Volterra type operator defined on this space.

Tomasz Małolepszy *Schröder equation and the existence of the blow-up solutions of some class of Volterra integral equations*

We show how the Schröder equation can be used to determine if the following Volterra integral equation

$$u(t) = \int_0^t k(t-s)g(u(s))\,ds, \qquad t \geq 0,$$

possesses a blow-up solution when $g(0) = 0$. We apply this approach to obtain the necessary and sufficient condition for the existence of the blow-up solutions in the following model of superdiffusion in the unbounded spatial domain of dimension N, $N = 1, 2, 3$:

$$\begin{aligned}
\frac{\partial}{\partial t}T(x,t) &= \sum_{n=1}^{N} \frac{\partial^\mu}{\partial|x_n|^\mu}T(x,t) + \lambda D(x|0)g(T(0,t)), \qquad x \in \mathbb{R}^N,\ t > 0,\\
T(x,0) &= 0, \qquad x \in \mathbb{R}^N,\\
\lim_{|x|\to\infty} T(x,t) &= 0, \qquad t > 0,
\end{aligned}$$

where the operator $\frac{\partial^\mu}{\partial|x_n|^\mu}$, $1 < \mu < 2$, is the so-called Riesz fractional derivative operator, the parameter of superdiffusion $\lambda > 0$ and the localization function $D(x|0)$ is defined as follows

$$D(x|0) = \begin{cases} 1, & x \in \Omega,\\ 0, & x \notin \Omega, \end{cases}$$

where $\Omega = \{x \in \mathbb{R}^N : \ -a < x_n < a\}$, $n = 1, 2, \ldots, N$, $0 < a \ll 1$.

[1] T. Małolepszy, *Nonlinear Volterra integral equations and the Schröder functional equation*, Nonlinear Anal. **74** (2011), 424–432.

[2] W.E. Olmstead, C.A. Roberts, *Dimensional influence on blow-up in a superdiffusive medium*, SIAM J. Appl. Math. **70** (2010), 1678–1690.

Janusz Matkowski *Continuity of means and uniqueness of invariant means*

In general the mean M need not to be continuous. It is known that *if* $M_i\colon I^p \to I$ *for* $i = 1, \ldots, p$ *are continuous means and*

$$\max(M_1\mathbf{x}), \ldots, M_p(\mathbf{x})) - \min(M_1(\mathbf{x}), \ldots, M_p(\mathbf{x})) < \max(\mathbf{x}) - \min(\mathbf{x}),$$

for all $\mathbf{x} = (x_1, \ldots, x_p) \in I^p$ *such that* $\min(\mathbf{x}) < \max(\mathbf{x})$, *then the sequence of iterates of the mean-type mapping* $\mathbf{M} = (M_1, \ldots, M_p)$ *converges to a mean-type* *mapping* $\mathbf{K} = (K, \ldots, K)$, *where* $K\colon I^p \to I$ *is a continuous and* $\mathbf{M}$*-invariant mean, i.e.* $K \circ \mathbf{M} = K$*; moreover, a continuous* $\mathbf{M}$*-invariant mean is unique.*

At this background it was an open and frequently asked question if there can exists another (necessarily discontinuous) $\mathbf{M}$-invariant mean.

We prove that the answer is "no". We also show that every increasing and homogeneous mean is continuous. Moreover, we give new conditions on convergence of the sequence of iterates of a mean-type mapping $\mathbf{M}$ to a unique $\mathbf{M}$-invariant mean-type mapping, where the continuity of the mean-type mapping is not assumed.

Bartosz Micherda *A new characterization of convex φ-functions with a parameter*

Let ρ be the Orlicz-Musielak modular generated by a φ-function with a parameter Φ, i.e. the functional of the form

$$\rho(f) = \int_{\Omega} \Phi(t, |f(t)|)\, d\mu(t),$$

and denote by L^{Φ} the corresponding Orlicz-Musielak space.

Then, as in metric spaces, for $D \subset L^{\Phi}$ we may define two operators: the projection onto D

$$P_D(f) = \Big\{ g \in D :\ \rho(f-g) = \inf_{d \in D} \rho(f-d) \Big\}$$

and the antiprojection onto D

$$P_D^a(f) = \Big\{ g \in D :\ \rho(f-g) = \sup_{d \in D} \rho(f-d) \Big\}, \quad \text{where } f \in L^{\Phi}.$$

In our talk, based on [2], we present a new theorem showing that, under some additional assumptions on the function Φ, all projections onto latticially closed subsets of L^{Φ} are isotonic (and all antiprojections onto such sets are antiisotonic) if and only if Φ is convex with respect to its second variable.

This gives the positive answer to the question presented as an open problem in [1].

[1] B. Micherda, *A characterization of convex φ-functions*, Opuscula Math. **32** (2012), 169–176.

[2] B. Micherda, *A new characterization of convex φ-functions with a parameter*, preprint.

Janusz Morawiec *Around a problem of Nicole Brillouët-Belluot*

Motivated by papers [1]–[5] we are interested in continuous bijections $f\colon I \to I$ satisfying

$$f(x)f^{-1}(x) = x^{\alpha} \qquad \text{for } x \in I,$$

where $I \subset (0, +\infty)$ is a nontrivial interval and α is a real number.

[1] R. Anschuetz, H. Scherwood, *When is a function's inverse equal to its reciprocal?*, College Math. J. **27** (1997), 388–393.

[2] R. Cheng, A. Dasgupta, B.R. Ebanks, L.F. Kinch, L.M. Larson, R.B. McFadden, *When does $f^{-1} = 1/f$?*, Amer. Math. Monthly **105** (1998), 704–716.

[3] R. Euler, J. Foran, *On functions whose inverse is their reciprocal*, Math. Mag. **54** (1981), 185–189.

[4] W. Jarczyk, J. Morawiec, *Note on an equation occurring in a problem of Nicole Brillouët-Belluot,* Aequationes Math. **84** (2012), 227–233.

[5] J. Morawiec, *On a problem of Nicole Brillouët-Belluot,* Aequationes Math. 84 (2012), 219–225.

Kazimierz Nikodem *Strongly convex set-valued maps*
(joint work with **H. Leiva, N. Merentes** and **J.L. Sánchez**)

Let $(X, \|\cdot\|)$ and $(Y, \|\cdot\|)$ be real normed spaces, D be a convex subset of X and B be the closed unit ball in Y. A set-valued map $F\colon D \to n(Y)$ is said to be *strongly convex with modulus* $c > 0$ if

$$tF(x_1) + (1-t)F(x_2) + ct(1-t)\|x_1 - x_2\|^2 B \subset F(tx_1 + (1-t)x_2),$$

for all $x_1, x_2 \in D$ and $t \in [0,1]$.

F is *strongly midconvex with modulus* $c > 0$ if

$$\frac{1}{2}F(x_1) + \frac{1}{2}F(x_2) + \frac{c}{4}\|x_1 - x_2\|^2 B \subset F\Big(\frac{x_1 + x_2}{2}\Big),$$

for all $x_1, x_2 \in D$.

Some properties of strongly convex (midconvex) set-valued maps are presented. In particular, a Bernstein-Doetsch and Sierpiński-type theorems for strongly midconvex set-valued maps, as well as a Kuhn-type result are obtained. A representation of strongly convex set-valued maps in inner product spaces and a characterization of product spaces involving this representation are given. Jensen and Hermite-Hadamard-type inequalities are proved. Finally, a connection between strongly convex set-valued maps and strongly convex sets is presented.

Agata Nowak *On Taylor reminder mean*

Let $I \subseteq \mathbb{R}$ be an open interval and $n \in \mathbb{N}$ be fixed. We consider a Taylor reminder mean of degree n given by

$$M_n^{[f]}(x,y) = \begin{cases} (f^{(n)})^{-1}\Big(n!\dfrac{f(y) - \sum_{k=0}^{n-1} \frac{f^{(k)}(x)}{k!}(y-x)^k}{(y-x)^n}\Big), & x \neq y \\ x, & x = y. \end{cases}$$

We discuss comparability problem for this class of means.

Andrzej Olbryś *On some inequalities equivalent to the Wright convexity*

Let $D \subset X$ be an open and convex subset of a real linear topological space. We say that a function $f\colon D \to \mathbb{R}$ is convex in the Wright sense if

$$\forall_{x,y\in D}\forall_{\lambda\in[0,1]}\ f(\lambda x + (1-\lambda)y) + f((1-\lambda)x + \lambda y) \le f(x) + f(y). \tag{1}$$

The aim of this talk is to give some inequalities which are equivalent to the inequality (1).

Zsolt Páles *Comparison of the geometric mean with Gini means*

For $p, q \in \mathbb{R}$, the *Gini mean* $G_{p,q}\colon \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \to \mathbb{R}_+$ is defined by

$$G_{p,q}(x_1,\dots,x_n) := \begin{cases} \left(\frac{x_1^p+\dots+x_n^p}{x_1^q+\dots+x_n^q}\right)^{\frac{1}{p-q}}, & \text{if } p \ne q,\\ \exp\left(\frac{x_1^p \ln(x_1)+\dots+x_n^p \ln(x_n)}{x_1^p+\dots+x_n^p}\right), & \text{if } p = q. \end{cases}$$

The general comparison problem of Gini means can be formulated in the following way: Given a nonempty subset $N \subseteq \mathbb{N}$, find necessary and sufficient conditions on the parameters $p, q, r, s \in \mathbb{R}$ such that the comparison inequality

$$G_{p,q}(x_1,\dots,x_n) \le G_{r,s}(x_1,\dots,x_n), \qquad n \in N,\ x_1,\dots,x_n \in \mathbb{R}_+$$

is satisfied.

The solution to this problem is known if $N = \mathbb{N}$ (see [1]) or if $N = \{2\}$ (see [2]).

In the main results, we give a complete answer to the above comparison problem for any subset N if either $(p,q) = (0,0)$ or $(r,s) = (0,0)$, i.e. if either $G_{p,q}$ or $G_{r,s}$ is the geometric mean.

[1] Z. Daróczy, L. Losonczi, *Über den Vergleich von Mittelwerten*, Publ. Math. Debrecen **17** (1970), 289–297 (1971).

[2] Zs. Páles, *Inequalities for sums of powers*, J. Math. Anal. Appl. **131** (1988), 265–270.

Boris Paneah *Asymptotic behavior of solutions to linear multi-dimensional functional equations depending on a small parameter and inverse problems*

The talk is devoted to the linear multi-dimensional functional operator

$$(\mathcal{P}F)(x) = \sum_{j=1}^{N} c_j(x)(F \circ a_j)(x), \qquad x \in D \subset \mathbb{R}^n.$$

Here $F \in C(I)$ with $I = (-1,1)$, and coefficients c_j and arguments a_j of $\mathcal{P}$ are sufficiently smooth functions $D \to \mathbb{R}$ and $D \to I$, respectively; D is a domain with a compact closure.

We will discuss the asymptotic behavior of solutions to equation $\mathcal{P}F = h_\varepsilon$ depending on a small parameter $\varepsilon \to 0$ under condition $h_\varepsilon = O(\varepsilon)$. This problem has been formulated by Ulam in his book “A collection of mathematical problems”, Los Alamos, 1941, in the case when $h_\varepsilon(x) = O(\varepsilon)$ for all $x \in D$, and in this form it generated an infinite stream of publications.

At the very beginning of this century it was established that in the original Ulam form the above problem is not well posed (in the Hadamard sense), as the input information $|\mathcal{P}F(x)| < \varepsilon$ for any $x \in D$ is redundant. For this reason the possibility to apply the results obtained have never been studied seriously. It turned out that in all considered cases (described in available to the author literature) the asymptotic behavior of a function F is determined completely by the validity of the inequality $|F(x)| < \varepsilon$ at the points x of some one-dimensional submanifold Γ (subject to determining), but not everywhere in D. The wide class of such operators $\mathcal{P}$ is considered in this talk, and, respectively, the asymptotic behavior of the solutions to equations $\mathcal{P}u = H_\varepsilon$. Our method make it possible to investigate in details a new (I hope) extremely interesting problem very important in different applications - inverse problem for the latter equation. This is a problem of reconstructing the form of operator $\mathcal{P}$, by using the asymptotic behavior of the solution to equation $\mathcal{P}F = H_\varepsilon$, $\varepsilon \to 0$.

Paweł Pasteczka *Arrow-Pratt index is instrumental in estimating differences among quasi-arithmetic means*

Quasi-arithmetic (QA) mean is defined for any continuous strictly monotone function $f\colon U \to \mathbb{R}$. We assume U to be open, bounded interval. When $\underline{a} = (a_1, \ldots, a_n)$ is a sequence of points in U and $\underline{w} = (w_1, \ldots, w_n)$ is a sequence of weights ($w_i > 0$, $w_1 + \ldots + w_n = 1$), then the mean $\mathfrak{M}_f(\underline{a}, \underline{w}) := f^{-1}(\sum_{i=1}^n w_i f(a_i))$ directly generalizing the way power means have been defined.

This family of means was shown, by Kolmogorov in 1930, to be very vast and ubiquitous. In fact, he proved that if a mean satisfies a tiny list of very natural axioms, then it has to be a QA mean for a certain function f.

Later, Mikusiński, in the first postwar issue of Banach and Steinhaus' renowned *Studia Mathematica*, put forward a very powerful tool in the theory of QA means. Namely, upon assuming $f, g \in \mathcal{C}^2(U)$ strictly monotone with nonvanishing first derivative

$$\mathfrak{M}_f \geq \mathfrak{M}_g, \text{ with equality only when the vector } \underline{a} \text{ is constant}$$

iff

$$\frac{f''}{f'} > \frac{g''}{g'} \text{ on a dense subset of } U.$$

This theorem makes "Mikusiński's mapping" $f \mapsto \frac{f''}{f'}$ most interesting to us (nowadays it is better known as the Arrow–Pratt measure of risk aversion or measure of absolute risk aversion).

Later, in 1960's, Cargo and Shisha obtained some majorizations of the difference $\|\mathfrak{M}_f - \mathfrak{M}_g\|_\infty$ uniform with respect to $\underline{a}$ and $\underline{w}$.

We are going, under the assumption of f and g being twice differentiable with the first derivative bounded away from zero, to word Cargo and Shisha's results in terms of Mikusiński's mapping. Then we relate the convergence of QA means in L_∞ norm to the convergence of the images of the underlying functions under a Mikusiński's mapping in L_1 norm.

Magdalena Piszczek *On selections of set-valued maps*

We present some applications of the result corresponding to the existence of a unique selection of a set-valued function satisfying inclusions in a single variable to the inclusions in several variables, especially the general linear inclusions or quadratic inclusions.

Wolfgang Prager *On a functional equation of O.G. Bokov*

Seizing a suggestion of Nicole Brillouët-Belluot (personal communication and Problem 4 posed at the 13th ICFEI), we consider the functional equation

$$f(x,y)f(x+y,z)+f(y,z)f(y+z,x)+f(z,x)f(z+x,y)=0,$$

introduced by O.G. Bokov in [1]. We present some solutions aside from those in $\mathbb{C}[[x,y]]$ given by A.V. Yagzhev in [2].

[1] O.G. Bokov, *A model of Lie fields and multiple-time retarded Green's functions of an electromagnetic field in dielectric media*, Nauchn. Tr. Novosib. Gos. Pedagog. Inst. **86** (1973), 3–9.

[2] A.V. Yagzhev, *A functional equation of theoretical physics*, Funct. Anal. Appl. **16** (1982), 38–44.

Barbara Przebieracz *Is dynamical system stable?*
(joint work with **Z. Moszner**)

Let $I\subset\mathbb{R}$ be an interval. A function $F\colon\mathbb{R}\times I\to I$ is called a *dynamical system* if it satisfies the translation equation

$$F(s,F(t,x))=F(s+t,x),\qquad s,t\in\mathbb{R},\ x\in I$$

and the identity condition

$$F(0,x)=x,\qquad x\in I.$$

Dynamical systems can be defined equivalently by other systems of equations. We consider stability in the sense of Hyers-Ulam of these systems, and of the translation equation in some classes of functions (in which the solution of the translation equation is a dynamical system).

Teresa Rajba *A generalization of multiple Wright-convex functions via randomization*

We define and study classes $\mathcal{W}_n(\Theta,\mathcal{M}_j)$ of non-negative real functions associated with the classes $\mathcal{M}_j$ of j-times monotone functions, where $j=0,1,2,\dots,\infty$ (see [2]). These classes are generalizations of n-Wright-convex functions introduced in [1].

For a fixed number $h\in\mathbb{R}$ the difference operator Δ_h, acting on a real function $F\colon\mathbb{R}\to\mathbb{R}$, is defined by $\Delta_hF(x)=F(x)-F(x-h)$ $(x\in\mathbb{R})$. The superposition of several difference operators will be denoted shortly $\Delta_{h_1h_2\dots h_n}$ $(h_1,h_2,\dots,h_n\in\mathbb{R},\ n\in\mathbb{N})$. The higher order convexity can be described in terms of difference operators $\Delta_{h_1 h_2 \dots h_{n+1}}$: a function F is *Wright-convex of order* n (or n-*Wright-convex*) if $\Delta_{h_1 h_2 \dots h_{n+1}} F(x) \geqslant 0$ for all $h_1, h_2, \dots, h_{n+1} > 0$.

Let Θ be a real valued random variable with the distribution concentrated on $[0, \infty)$. Replacing in $\Delta_{h_1 h_2 \dots h_n}$ the real numbers $h_1, \dots, h_n$ ($n \in \mathbb{N}$) by independent random variables $\Theta_1, \dots, \Theta_n$ with the same distribution as the random variable Θ, and taking expectation, we define the *randomized difference operator* $\Phi^n = \Phi^n_{\Theta_1 \dots \Theta_n}$ by

$$\Phi^n F(x) = \Phi^n_{\Theta_1 \dots \Theta_n} F(x) = E\Delta_{\Theta_1 \dots \Theta_n} F(x), \qquad x \in \mathbb{R}.$$

Given a function $F \in \mathcal{M}_j$ ($j = 0, 1, 2, \dots, \infty$), we say that F *is* n-*times* Θ-*Wright-convex with respect to* $\mathcal{M}_j$, or that $F \in \mathcal{W}_n(\Theta, \mathcal{M}_j)$, if

$$\Phi^k_{\Theta_1 \dots \Theta_k} F \in \mathcal{M}_j$$

for all $k = 1, 2, \dots, n$.

We prove that each function from $\mathcal{W}_n(\Theta, \mathcal{M}_j)$ can be represented as a series of functions generated by a function from $\mathcal{M}_j$. We give an integral representation of these functions in the case when the random variable Θ has an exponential or a discrete arithmetic distribution. As a consequence we show, that for an arithmetic discrete Θ,

$$\bigcap_{n=1}^{\infty} \mathcal{W}_n(\Theta, \mathcal{M}_j) \supsetneq \mathcal{M}_\infty$$

and that when the Θ is exponential we have equality in the above formula.

[1] A. Gilányi, Zs. Páles, *On convex functions of higher order*, Math. Inequal. Appl. **11** (2008), 271–282.

[2] T. Rajba, *A generalization of multiple Wright–convex functions via randomization*, J. Math. Anal. Appl. **388** (2012), 548–565.

Maciej Sablik *Additivity of insurance premium*

Let u denote a utility function, X be a random loss, $H(X)$ - a premium paid in case of loss, and, finally, let w denote the initial wealth of insurer. Then the generalized zero utility principle under the rank-dependent utility model may be expressed as the following equation

$$u(w) = E_g(u(w + H(X) - X)), \tag{1}$$

where $g: [0, 1] \to [0, 1]$ is a so called probability distortion function, and E_g denotes the Choquet integral. We ask for utility and probability distortion functions satisfying (1) if additionally the additivity of H for independent risks is assumed.

[1] S. Heilpern, *A rank–dependent generalization of zero utility principle*, Insurance Math. Econom. **33** (2003), 67–73.

[2] D. Kahneman, A. Tversky, *Prospect theory: an analysis of decision under risk*, Econometrica **47** (1979), 263–291.

[3] M. Kaluszka, M. Krzeszowiec, *Pricing insurance contracts under Cumulative Prospect Theory*, Insurance Math. Econom. **50** (2012), 159–166.

[4] A. Tversky, D. Kahneman, *Advances in prospect theory: cumulative representation of uncertainty*, J. of Risk and Uncertainty **5** (1992), 297–323.

Jens Schwaiger *Remarks on the history of some stability results*

Several reviews (Zbl 1256.39022, Zbl 1256.39019, Zbl 1245.39018, Zbl 1219.39020, Zbl 1219.39011) in *Zentralblatt MATH* by Gian Luigi Forti contain a remark saying that a generalization of Rassias' theorem due to Găvruţă is a special case of a result from 1980 by him. A review in the same journal (Zbl 1232.39026) by Găvruţă contains a remark saying that Forti's remark *clearly is not true.*

The results of both authors are discussed and compared. In doing so it is shown that in the speakers opinion Forti is right.

Ekaterina Shulman *On almost subadditive set-functions on groups*

Let G be a group and Ω be an arbitrary set. A map $F\colon G \to 2^{\Omega}$ is called subadditive if $F(gh) \subset F(g) \cup F(h)$ for all $g, h \in G$. Some stability type problems for subadditive functions will be discussed.

Andrzej Smajdor *Permutability of set-valued cosine families*
(joint work with **W. Smajdor**)

Let K be a closed convex cone with the nonempty interior in a real Banach space.

A one-parameter family $\{F_t : t \geq 0\}$ of multifunctions $F_t\colon K \to cc(K)$ is said to be *cosine* if

$$F_0 = I_d$$

and

$$F_{t+s} + F_{t-s} = 2F_t \circ F_s,$$

whenever $0 \leq s \leq t$.

A cosine family $\{F_t : t \geq 0\}$ is *regular* if

$$\lim_{t \to 0+} d(F_t(x), \{x\}) = 0$$

for every $x \in K$.

Theorem
If $\{F_t : t \geq 0\}$ is a regular cosine family of continuous additive set-valued functions $F_t\colon K \to cc(K)$ such that $x \in F_t(x)$ for $t \geq 0$ and $x \in K$, then

$$F_t \circ F_s = F_s \circ F_t$$

for $s, t \geq 0$.

M. Sova in 1966 proved that every regular cosine family of single-valued continuous linear functions on a Banach space is permutable. The goal of this paper is to prove the same for regular cosine families of continuous additive set-valued functions.

Wei Song *The first order iteration problem for iterative equations*

By mean of fixed point theorems, many scholars have discussed the general iterative equation $H(f(x), f^2(x), \ldots, f^n(x)) = F(x)$, $n \geq 2$. Because of difficulties in applying fixed point theorems, most of known results on solutions demand that the first order iteration of the unknown function f must appear in these equations. So we name this phenomenon the first order iterative problem. In the present paper we will discuss C^1 solutions for the general iterative equations without the assumption that the first order iterations of an unknown function f must appear.

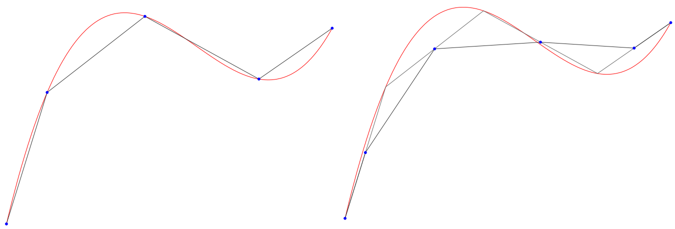
Peter Stadler *Curve shortening by short rulers*

We look at homomorphisms $h\colon (\mathbb{R},+) \to (G,\circ)$ on a Lie group G:

$$h(s+t) = h(s) \circ h(t), \quad h(0) = e \quad \text{and} \quad h(1) = g.$$

The restriction of h to the interval $[0,1]$ is a geodesic, i.e. a locally shortest line.

The problem is to construct long geodesics. But any curve connecting two points can be shortened by using a ruler which allows to construct short geodesics.



In normed vector spaces, the curve converges to the straight line if it's shortened iterative. This result can be generalized to planes which are curved in one direction.

Henrik Stetkær *An exponential-cosine functional equation*

Let $(G,+)$ be an abelian group and let $\mu\colon G \to \mathbb{C}$. The proposition below extends results by Parnami, Singh and Vasudeva [1] to the equation

$$g(x+y) + \mu(y)g(x-y) = 2g(x)g(y), \qquad x,y \in G, \tag{1}$$

in which $g\colon G \to \mathbb{C}$ is the unknown function. G is a Banach space in [1].

Proposition
Let g be a solution of (1) such that $g(0) = 1$.

1. *If $G = 2G$, then μ is multiplicative from G to $\mathbb{C}^*$.*
2. *If $\mu\colon G \to \mathbb{C}^*$ is multiplicative, then g has the form*

$$g(x) = \frac{\chi(x) + \mu(x)\chi(-x)}{2} \qquad \text{for all } x \in G$$

for some multiplicative map $\chi\colon G \to \mathbb{C}^*$.

For more information see Chapter 9 of the forthcoming book [2].

[1] J.C. Parnami, H. Singh, H. L. Vasudeva, *On an exponential-cosine functional equation*, Period. Math. Hungar. **19** (1988), 287–297.

[2] H. Stetkær, *Functional Equations on Groups*, World Scientific Publishing Co. 2013.

Stevo Stević *Behaviour at infinity of solutions of some linear functional and functional-difference equations*

We present some results on the linear functional equation

$$x(\phi(t)) = \alpha(t)x(t) + f(t), \qquad t \in \mathbb{R} \text{ (or } t \in \mathbb{C}),$$

as well as on the linear functional-difference equation

$$x(\phi(n)) = \alpha(n)x(n) + f(n), \qquad n \in \mathbb{Z},$$

under some conditions posed on functions α, ϕ and f.

Some applications of these results on the behavior of bounded at infinity solutions of the functional equation

$$x(\phi^{[k]}(t)) = \sum_{j=0}^{k-1} \alpha_j(t)x(\phi^{[j]}(t)) + f(t), \qquad t \in \mathbb{R} \text{ (or } t \in \mathbb{C}),$$

as well as of the functional-difference equation

$$x(\phi^{[k]}(n)) = \sum_{j=0}^{k-1} \alpha_j(n)x(\phi^{[j]}(n)) + f(n), \qquad n \in \mathbb{Z},$$

under some conditions posed on functions α_j, $j = 0, 1, \ldots, k-1$, ϕ and f, are also presented.

Tomasz Szostok *Functional equations connected with quadrature rules of Hermite and Birkhoff*

In classical quadrature rules of numerical analysis the definite integral of a given function is approximated by a weighted sum of values of f. As it is well known for polynomials of certain degree this approximation gives exact results. Thus the following equation

$$F(y) - F(x) = (y-x)[a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)]$$

becomes interesting. In the current talk we consider a functional equation connected with quadrature rules of Hermite and Birkhoff. In the Hermite quadrature rule the integral of f is approximated with use of values of f (taken from the interval of integration) and values of f' at the endpoints. Therefore we get the equation

$$\begin{aligned} F(y) - F(x) = (y-x)[a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)] \\ + (y-x)^2[g(y) - g(x)]. \end{aligned} \tag{1}$$

Similarly, in Birkhoff rule values of f'' are used which yields the equation

$$\begin{aligned} F(y) - F(x) = (y-x)[a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)] \\ + (y-x)^3[b_1 g(\gamma_1 x + \delta_1 y) + \ldots + b_k g(\gamma_k x + \delta_k y)]. \end{aligned} \tag{2}$$

We obtain solutions of equations (1) and (2) (under some assumptions on the coefficients ocurring in these equations).

Patricia Hilario Tacuri *Existence and uniqueness of solution of measure neutral functional differential equations*

We introduce the equations called neutral measure functional differential equations and we prove that these equations can be also related with a class of abstract generalized ordinary differential equations (GODEs). Then, using this correspondence, we are able to prove existence and uniqueness of solutions and continuous dependence results for neutral measure functional differential equations.

Jörg Tomaschek *On the characterization of generalized Dhombres functional equations*
(joint work with **L. Reich**)

The generalized Dhombres functional equation in the complex domain is given by

$$f(zf(z)) = \varphi(f(z)),$$

where f is an unknown function and φ is a known one.

At ICFEI 14th a talk on the solvability of this equation for non constant formal or local analytic solutions f, where $f(0) = w_0 \in \mathbb{C} \setminus \{0\}$, was presented. In this talk we continue these investigations for the case where $f(\infty) = w_0 \in \mathbb{C} \setminus \{0\}$ or $f(\infty) = \infty$ and $f(z_0) = 1$ for $z_0 \neq 0$.

[1] L. Reich, J. Smítal, M. Štefánková, *Local analytic solutions of the generalized Dhombres functional equations I*, Österreich. Akad. Wiss. Math.-Natur. Kl. Sitzungsber. II **214** (2005), 3–25.

[2] L. Reich, J. Smítal, M. Štefánková, *Local analytic solutions of the generalized Dhombres functional equations II*, J. Math. Anal. Appl. **355** (2009), 821–829.

[3] J. Tomaschek, *Contributions to the local theory of generalized Dhombres functional equations in the complex domain*, Grazer. Math. Ber. **358**, p. 72+iv (2011).

[4] J. Tomaschek, L. Reich, *Local solutions of the generalized Dhombres functional equation in a neighbourhood of infinity*, submitted.

Peter Volkmann *On stability of* $\max\{f((xy)y), f(x)\} = f(xy) + f(y)$
(joint work with **R. Badora** and **B. Przebieracz**)

Let S be a groupoid having a left unit, and suppose for $x, y \in S$ there always exists $k \in \{1, 2, 3, \ldots\}$ such that

$$(xy)^{2^k} = x^{2^k} y^{2^k}, \qquad ((xy)y)^{2^k} = (x^{2^k} y^{2^k})y^{2^k}, \tag{1}$$

the powers x^{2^k} being recursively defined. We consider the functional equation given in the title for real-valued functions f defined on S, and we show its stability in the sense of Pólya-Szegő-Hyers-Ulam. The special case $(xy)^2 = x^2y^2$ of (1) had been treated in 2011 by A. Gilányi, Kaori Nagatou and P. Volkmann.

Jacek Wesołowski *Tail asymptotics for random perpetuities*

We will consider the random perpetuity equation

$$R \stackrel{d}{=} RM + Q, \tag{1}$$

where R, M, Q are real random variables, R and the pair (M, Q) are independent. Here $\stackrel{d}{=}$ denotes equation in law. Under suitable conditions on the joint distribution of (M, Q) this equation (1) has a unique (in distribution) probabilistic solution, being the law of the random variable R. Note that (1) can be rewritten as

$$\int\limits_{\mathbb{R}} f(u)\, \nu(du) = \int\limits_{\mathbb{R}^2} \int\limits_{\mathbb{R}} f(au + b)\, \nu(du)\, \mu(da, db),$$

for any function f which is continuous and has a compact support, where μ denotes the distribution of (M, Q) (assumed to be known) and ν is the unknown distribution of R.

Investigation of tail behaviour of the measure ν (which in, so called, critical case may not even be probabilistic - see e.g. Buraczewski (2007)) goes back to the seminal paper of Kesten (1973). Thin tails were considered e.g. in Goldie and Grübel (1996). The results I will present are concerned with even thinner tails.

[1] D. Buraczewski, *On invariant measures of stochastic recursions in a critical case*, Ann. Appl. Probab. **17** (2007), 1245–1272.

[2] C.M. Goldie, R. Grübel, *Perpetuities with thin tails*, Adv. Appl. Probab. **28** (1996), 463–480.

[3] P. Hitczenko, J. Wesołowski, *Perpetuities with thin tails, revisited*, Ann. Appl. Probab. **19** (2009), 2080–2101.

[4] H. Kesten, *Random difference equations and renewal theory for products of random matrices*, Acta Math. **131** (1973), 207—248.

Alfred Witkowski *Invariance equation for means of power growth*

A symmetric, homogeneous mean M is called a mean of power growth if there exists a number m called order of M such that the limit $\lim_{x \to 0+} M(x, 1)/x^m$ is positive.

We show that if, under some weak assumptions, the invariance equation $M(N, K) = M$ has a solution in the class of means of power growth, then the orders of M, N, K must be equal.

As an application we conclude that the invariance equation has only the trivial solution $M = N = K$ in the class of Heinz means.

Paweł Wójcik *Linear operators preserving orthogonality*

Let H, K be Hilbert spaces. For $\varepsilon \in [0, 1)$, we define *approximate orthogonality* of vectors x and y

$$x \perp^{\varepsilon} y \;:\Longleftrightarrow\; |\langle x|y\rangle| \leqslant \varepsilon \|x\| \cdot \|y\|.$$

We say that $f \in L(H;K)$ is *approximately orthogonality preserving* iff

$$\forall_{x,y\in H}\ x\perp^{\delta} y \implies fx\perp^{\varepsilon} fy$$

with some $\delta, \varepsilon \in [0,1)$. In particular, if the linear operator f satisfies this statement with $\delta = \varepsilon$, then f has to be a similarity, i.e. a scalar multiple of an isometry. We will discuss the problem, whether each approximately orthogonality preserving operator $f \in L(H;K)$ can be approximated by an orthogonality preserving operator $h \in L(H;K)$, i.e.

$$\forall_{x,y\in H}\ x\perp y \implies hx\perp hy.$$

Marek Cezary Zdun *On commuting continuous mappings nonembeddable in iteration semigroups*
(joint work with **D. Krassowska**)

Let $f, g: I = (0,b] \to I$ be commuting continuous injections. We consider the case when there is no semigroup in which f and g can be embedded. We explain the reasons of this phenomenon and modify the definition of an iteration semigroup introducing a new notion a refinement iteration semigroup that is a family $\{f^t: I \to I,\ t \in T\}$ for which $f^t \circ f^s = f^{t+s}$, $t, s \in T$, such that $f = f^1$ and $g = f^s$ for an $s \in T$, where $T \subsetneq \mathbb{R}^+$ is a dense in $\mathbb{R}^+$ additive semigroup. We determine a wide class of semigroups T admitting the embeddability of f and g.

Problems and Remarks

1. Problem.

Let X, Y be the normed spaces, U be a nonempty subset of X. We say that a function $f: U \to Y$ is *Jensen on* U if it is satisfies

$$f\Big(\frac{x+y}{2}\Big) = \frac{f(x)+f(y)}{2}, \qquad x, y \in U,\ \frac{x+y}{2} \in U. \tag{1}$$

We present some hyperstability result for the equation (1). Namely, for some natural particular forms of φ (and under some additional assumptions on U), the conditional functional equation (1) is φ-hyperstable in the class of functions $f: U \to Y$, i.e. each $f: U \to Y$ satisfying the inequality

$$\Big\| f\Big(\frac{x+y}{2}\Big) - \frac{f(x)+f(y)}{2} \Big\| \le \varphi(x,y), \qquad x, y \in U,\ \frac{x+y}{2} \in U,$$

must be Jensen on U. The following result was proved in the paper [1].

Theorem
Let U be nonempty subset of $X \setminus \{0\}$ and $f: U \to Y$ satisfies

$$\Big\| f\Big(\frac{x+y}{2}\Big) - \frac{f(x)+f(y)}{2} \Big\| \le c\|x\|^p\|y\|^q, \qquad x, y \in U,\ \frac{x+y}{2} \in U, \tag{2}$$

where $c \ge 0$, $p, q \in \mathbb{R}$. Assume that the set U and the numbers p, q satisfy one of the following conditions:

(a) $p+q<0$, *and there exists a positive integer* n_0 *with*

$$nx \in U, \qquad x \in U,\ n \in \mathbb{N},\ n \geq n_0,$$

(b) $p+q>1$, *and there exists a positive integer* n_0 *with*

$$-\frac{1}{n}x, \frac{1}{2}\Big(1-\frac{1}{n}\Big)x \in U, \qquad x \in U,\ n \in \mathbb{N},\ n \geq n_0,$$

(c) $0<p+q<1$, *there exists a positive integer* n_0 *with*

$$\frac{1}{n}x, \Big(2-\frac{1}{n}\Big)x \in U, \qquad x \in U,\ n \in \mathbb{N},\ n \geq n_0.$$

Then f *is Jensen on* U.

It is also known that the additional assumptions on U in above theorem are necessary and we do not have the hyperstability of the Jensen equation on U if $p+q=1$.

For the case $p+q=0$, the method used in the proof of the above Theorem can not be applied, thus this is still an open problem. However, if $p=q=0$ ($\varphi(x,y)=c$), the case was investigated, (see for example [2, 3]) and then the equation (1) is not φ-hyperstable in the class of functions $f\colon U \to Y$.

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[2] S.-M. Jung, *Hyers-Ulam-Rassias stability of Jensen's equation*, Proc. Amer. Math. Soc., **126** (1998), 3137–3143.

[3] S.-M. Jung, M.S. Moslehian, P.K. Sahoo, *Stability of a generalized Jensen equation on restricted domains*, J. Math. Ineq., **4** (2010), 191–206.

Anna Bahyrycz

2. Problem.

Let f be a complex-valued function on the semigroup S. If f satisfies

$$\begin{aligned} f(xyz)+f(xzy)+f(yzx)+f(zxy)+f(zyx) \\ = 3f(x)[f(yz)+f(zy)]+3f(y)[f(zx)+f(xz)] \\ +3f(z)[f(xy)+f(yx)]-12f(x)f(y)f(z) \end{aligned}$$

for all x,y,z in S must be CENTRAL? That is must f satisfy

$$f(xy)=f(yx)$$

for all x,y in S?

Thomas M.K. Davison

3. Remark.

If $I \subset \mathbb{R}$ is an interval and $f, g\colon I \to \mathbb{R}$ are continuous increasing functions such that $f+g$ is strictly increasing, then $A^{[f,g]}\colon I^2 \to I$ defined by

$$A^{[f,g]}(x,y) := (f+g)^{-1}(f(x)+g(y)), \qquad x,y \in I,$$

is a mean generalizing the weighted quasi-arithmetic mean ([4] cf. also [2], [3]).

The following result contains an invariance formula for this type of means.

Theorem

Let $f, g, h\colon I \to \mathbb{R}$ be continuous functions. Suppose that f, g are strictly increasing and h, $f-h$, $g-h$ are increasing. Then:

(i) *the mean $A^{[f,g]}$ is invariant with respect to the mean-type mapping $(A^{[f-h,h]}, A^{[h,g-h]})\colon I^2 \to I^2$ ([1]), that is*

$$A^{[f,g]} \circ (A^{[f-h,h]}, A^{[h,g-h]});$$

(ii) *the sequence of iterates $((A^{[f-h,h]}, A^{[h,g-h]})^n)_{n\in\mathbb{N}}$ converges in I^2,*

$$\lim_{n\to\infty} (A^{[f-h,h]}, A^{[h,g-h]})^n = (A^{[f,g]}, A^{[f,g]}),$$

(and the limit does not depend on the function h);

(iii) *a function $\Phi\colon I^2 \to \mathbb{R}$, being continuous on the diagonal $\{(x,x) : \ x \in I\}$, satisfies the functional equation*

$$\Phi(A^{[f-h,h]}(x,y), A^{[h,g-h]}(x,y)) = \Phi(x,y), \qquad x,y \in I,$$

if, and only if, there is a continuous function $\varphi\colon I \to \mathbb{R}$ such that

$$\Phi(x,y) = \varphi(A^{[f,g]}(x,y)), \qquad x,y \in I.$$

If $k \in \mathbb{N}$, $k \geq 2$, and $f_1, \ldots, f_k\colon I \to \mathbb{R}$ are continuous, increasing functions such that $f_1 + \ldots + f_k$ is strictly increasing, then $A^{[f_1,\ldots,f_k]}\colon I^k \to I$,

$$A^{[f_1,\ldots,f_k]}(x_1,\ldots,x_k) := (f_1+\ldots+f_k)^{-1}(f_1(x_1)+\ldots+f_k(x_k)), \qquad x_1,\ldots,x_k \in I,$$

generalizes the k-variable weighted quasi-arithmetic mean ([4]).

The above result can be extended to the class of k-variable means.

[1] J. Matkowski, *Invariant and complementary quasi-arithmetic means*, Aequationes Math. **57** (1999), 87–107.

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[3] J. Matkowski, P. Volkmann, *A functional equation with two unknown functions*, http://www.uni-karlsruhe.de/~semlv, Seminar LV, No. 30, 6 pp., 28.04.2008.

[4] J. Matkowski, *Generalized weighted quasi-arithmetic means*, Aequationes Math. **79** (2010), 203–212.

Janusz Matkowski

4. Problem.

Given an open interval I, functions $f\colon I \to \mathbb{R}$ of the form $f = g - h$, where $g, h\colon I \to \mathbb{R}$ are nondecreasing functions, are characterized by the property that they are of bounded variation on any compact subinterval of I, that is, for any $[a,b] \subseteq I$,

$$V_{[a,b]}f := \sup\left\{ \sum_{i=1}^{n} |f(t_i) - f(t_{i-1})| : \ (t_0, t_1, \dots, t_n) \in \mathcal{P}_{[a,b]} \right\}$$

is finite. Here $\mathcal{P}_{[a,b]}$ denotes the set of partitions of the interval $[a,b]$ defined by

$$\mathcal{P}_{[a,b]} := \bigcup_{n=1}^{\infty} \{(t_0, t_1, \dots, t_n) : \ a = t_0 < t_1 < \dots < t_n = b\}.$$

This remarkable result of Jordan was extended to convex differences by Frigyes Riesz. He proved that $f\colon I \to \mathbb{R}$ is of the form $f = g - h$, where $g, h\colon I \to \mathbb{R}$ are convex functions if and only if f has bounded second-order variation on any compact subinterval of I, that is, for any $[a,b] \subseteq I$,

$$V^2_{[a,b]}f := \sup\left\{ \sum_{i=1}^{n-1} \left| \frac{f(t_i) - f(t_{i-1})}{t_i - t_{i-1}} - \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i} \right| : \ (t_0, t_1, \dots, t_n) \in \mathcal{P}_{[a,b]} \right\}.$$

Now consider the problem of characterizing Jensen convex differences, i.e. functions of the form $f = g - h$, where $g, h\colon I \to \mathbb{R}$ are Jensen convex functions. For these functions, $V^2_{[a,b]}f$ is not finite in general. However, one can verify that if f is a Jensen convex difference, then the following second-order $\mathbb{Q}$-variation is finite:

$$V^{2,\mathbb{Q}}_{[a,b]}f := \sup\left\{ \sum_{i=1}^{n-1} \left| \frac{f(t_i) - f(t_{i-1})}{t_i - t_{i-1}} - \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i} \right| : \ (t_0, t_1, \dots, t_n) \in \mathcal{P}^{\mathbb{Q}}_{[a,b]} \right\},$$

where $\mathcal{P}^{\mathbb{Q}}_{[a,b]}$ denotes the set of $\mathbb{Q}$-partitions of the interval $[a,b]$ defined by

$$\mathcal{P}^{\mathbb{Q}}_{[a,b]} := \left\{ (t_0, t_1, \dots, t_n) \in \mathcal{P}_{[a,b]} : \ \frac{t_i - a}{b - a} \in \mathbb{Q}, \ (i = 1, \dots, n-1) \right\}.$$

The open problem is to show the reversed implication, that is, the finiteness of $V^{2,\mathbb{Q}}_{[a,b]}f$ for every $[a,b] \subseteq I$ implies that f is of the form $f = g - h$, where $g, h\colon I \to \mathbb{R}$ are Jensen convex functions.

Zsolt Páles

5. Problem.

Given an open interval I and a strictly increasing continuous function f, the quasi-arithmetic mean M_f is defined by

$$M_f(x_1, \dots, x_n) := f^{-1}\Big(\frac{x_1 + \dots + x_n}{n}\Big), \qquad n \in \mathbb{N},\ x_1, \dots, x_n \in I.$$

For a twice differentiable function f with a nonvanishing first derivative, the Arrow–Pratt index of the mean M_f is defined as $A_f := f''/f'$. The distance of two quasi-arithmetic means M_f and M_g was defined by Paweł Pasteczka in his talk by

$$\rho(M_f, M_g) = \sup\{|M_f(x_1, \dots, x_n) - M_g(x_1, \dots, x_n)| : \ n \in \mathbb{N}, x_1, \dots, x_n \in I\}.$$

Pasteczka gave an upper estimate on this distance using the Arrow–Pratt index of M_f and M_g provided that f and g are twice differentiable functions with a nonvanishing first derivatives.

One can be interested in an upper estimate for $\rho(M_f, M_g)$ which does not involve the second order derivatives of f and g. A three variable function which is naturally connected to the quasi-arithmetic mean M_f is the function μ_f given by

$$\mu_f(x, y, u) := \frac{f(u) - f(x)}{f(y) - f(x)}, \qquad x \le u \le y, \, x < y, \, x, y \in I.$$

Using this function, one can, for instance, show that

(a) $M_f \le M_g$ if and only if $\mu_f \le \mu_g$;

(b) A sequence M_{f_n} converges pointwise to M_f if and only if μ_{f_n} converges pointwise to μ_f.

The open problem is to establish a formula or an upper estimate for the distance $\rho(M_f, M_g)$ in terms of μ_f and μ_g.

Zsolt Páles

6. Problem.

The talk is devoted to the linear multi-dimensional functional operator

$$(\mathcal{P}F)(x) = \sum_{j=1}^{N} c_j(x)(F \circ a_j)(x), \qquad x \in D \subset \mathbb{R}^n.$$

Here $F \in C(I)$ with $I = \{t \mid \ -1 \le t \le 1\}$, and coefficients c_j and arguments a_j of $\mathcal{P}$ are sufficiently smooth functions $D \to \mathbb{R}$ and $D \to I$, respectively; D is a domain with a compact closure.

We will discuss the asymptotic behavior of solutions to equation $\mathcal{P}F = h_\varepsilon$ depending on a small parameter $\varepsilon \to 0$ under condition $h_\varepsilon = O(\varepsilon)$. This problem has been formulated by Ulam in his book "A collection of mathematical problem", Los Alamos, 1941, in the case when $h_\varepsilon(x) = O(\varepsilon)$ for all $x \in D$.

At the very beginning of this century it was established that in the original Ulam form the above problem is not well posed (in the Hadamard sense), as the input information ($|\mathcal{P}F(x)| < \varepsilon$ for any $x \in D$) is redundant. It turned out that in all considered cases the asymptotic behavior of a function F is determined completely by the validity of the latter inequality only at the points x of some one-dimensional submanifold $\Gamma \subset D$ (subject to determining), but not everywhere in D.

Turn out to the examples of functional operators.

1. The Cauchy type functional operators. The functional operator

$$(\mathfrak{C}F)(x) = F(a(x)) - \sum_{j=1}^{N} F \circ a_j(x), \qquad x \in D \subset \mathbb{R}^n, \tag{1}$$

with

$$a = \sum_{j=1}^{N} a_j \qquad \text{everywhere in } D. \tag{2}$$

has never been studied with the asymptotic point of view with the exception of, probably, linear functions $a(x)$, $a_j(x)$. If (2) holds only at points x of a curve $\Gamma \subset D$, then the operator $\mathfrak{C}$ is called *weak Cauchy type operator* (*along* Γ).

Proposition 1

Let $\mathfrak{C}$ be a weak Cauchy operator (1) along $\Gamma \subset D$. Then there is a constant c (depending on Γ) such that any solution F of the equation $\mathfrak{C}(F) = H$ with

$$|H_\Gamma|_{\langle r \rangle} < \varepsilon$$

satisfies the condition

$$F(t) = \lambda t + c\varepsilon$$

for some real λ.

2. Quasiquadratic functional operators. Turn now to a little studied quasi-quadratic operators, for example,

$$\mathcal{Q}(F) := F(x_1 + x_2) + F(x_1 - x_2) - c_1 F(x_1) - c_2 F(x_2),$$

in $D = \{x \mid \ |x_1 \pm x_2| \leqslant 1\}$, c_1, $c_2 > 0$.

Take a curve $\Gamma = \{x \mid \ x_1 = t,\, x_2 = t + 1;\ -1 \leqslant t \leqslant 0\}$.

Proposition 2

1°. *If $c_1 + c_2 \neq 2^k$ for any integer k, $k \geq 2$, then $F = 0$ is the unique solution of the equation $\mathcal{Q}_\Gamma F = 0$ in the space C^m, $m = \lceil \log_2(c_1 + c_2) \rceil$.*

2°. *If $c_1 + c_2 = 2^m$ and $F \in C^m$, $\mathcal{Q}_\Gamma F = 0$, then $F = \sum_{j=0}^{m} a_j t^j$ with $\overline{a} = (a_0, a_1, \ldots, a_m)$ a vector from the subspace $\ker \Lambda_m$.*

But if $|\mathcal{Q}_\Gamma F|_r < \varepsilon$ for all arbitrary small $\varepsilon > 0$, then $F(t) = \sum_{j=0}^{m} a_j t^j + O(\varepsilon)$ $0 \leqslant t \leqslant 1$.

As to Λ_m, it is the matrix of the operator $\mathcal{Q}_\Gamma$ in the space of polynomials with the basis $1, t, \ldots, t^m$.

Boris Paneah

7. Problem.

Let $\mathfrak{M}_f$ denotes a quasi-arithmetic mean generated by a function $f\colon I \to \mathbb{R}$, I – an open interval. Let $A(f) := f''/f'$ and $(f_n)_{n\in\mathbb{N}}$ be a family of functions $f_n \in \mathcal{C}^2(I)$, $f_n' \neq 0$ satisfying

$$A(f_1)(x) \leq A(f_2)(x) \leq \dots \qquad \text{for all } x \in I.$$

One can ask how to express the property

$$\mathfrak{M}_{f_n} \to \max \text{ pointwise} \tag{1}$$

in terms of operator A. In [1] it was proved that

$$\text{(1) implies } A(f_n) \to \infty \text{ on some dense subset of } I, \tag{2}$$

$$A(f_n) \to \infty \text{ on the whole interval } I \text{ implies (1).} \tag{3}$$

On 15th ICFEI it was announced that the implication (3) might be strengthened to

$$A(f_n) \to \infty \text{ a.e. on } I \text{ implies (1).} \tag{4}$$

There appeared a natural question. How to fulfilled a gap between conditions (2) and (4)? Namely, how to express a necessary and sufficient condition of (1) in terms of operator A?

Remark 1
Upon replacing ∞, $\leq$ and max by $-\infty$, $\geq$ and min respectively one gets a dual, and equivalent, problem.

Remark 2
This problem is closely related to the one presented in [1, p.207].

[1] P. Pasteczka, *When is a Family of Generalized Means a Scale?*, Real Anal. Exchange, **38** (2013), 193–210.

Paweł Pasteczka

8. Problem.

We present some stability and hyperstability results for the Drygas equation on restricted domain. Let X be a nonempty subset of a normed space and Y be a normed space. We say that a function $f\colon X \to Y$ satisfies the Drygas functional equation on X if

$$f(x+y) + f(x-y) = 2f(x) + f(y) + f(-y), \quad x, y \in X,\ x+y, x-y \in X.$$

Theorem
Let X be a nonempty subset of a normed space, Y be a Banach space, $c \geq 0$ and a function $f\colon X \to Y$ satisfy

$$\|f(x+y) + f(x-y) - 2f(x) - f(y) - f(-y)\| \leq c(\|x\|^p + \|y\|^p)$$

for all $x, y \in X$ such that $x+y, x-y \in X$.

1) *If $p > 2$, X is such that $0, -x, \frac{x}{2} \in X$ for all $x \in X$, then there exists a unique function $g\colon X \to Y$ satisfying the Drygas equation on X such that*

$$\|f(x) - g(x)\| \leq \frac{2c}{2^p - 4}\|x\|^p, \qquad x \in X.$$

2) *If $0 < p < 1$, X is such that $0, -x, 2x \in X$ for all $x \in X$, then there exists a unique function $g\colon X \to Y$ satisfying the Drygas equation on X such that*

$$\|f(x) - g(x)\| \leq \frac{2c}{2 - 2^p}\|x\|^p, \qquad x \in X.$$

3) *If $p = 0$, X is such that $0, -x, 2x, 3x \in X$ for all $x \in X$, then there exists a function $g\colon X \to Y$ satisfying the Drygas equation on X such that*

$$\|f(x) - g(x)\| \leq c, \qquad x \in X.$$

4) *If $p < 0$, X is such that $0 \notin X$ and there exists $n_0 \in \mathbb{N}$ with $nx \in X$ for $x \in X$, $n \in \mathbb{N}$, $n \geq n_0$, then f satisfies the Drygas equation on X.*

The open problem is: what happened if $p \in [1, 2]$?

Magdalena Piszczek

9. Remark.

The following functional equation is connected with quadrature rules of numerical integration

$$F(y) - F(x) = (y - x)[a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)]. \tag{1}$$

In [1] it was (under some assumptions) proved that if $f, F\colon \mathbb{R} \to \mathbb{R}$ satisfy (1) then F must be continuous. This means that the (possibly) discontinuous part of f vanishes at the right hand side and, consequently, the expression

$$a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y)$$

is continuous.

Recently, more general equations stemming from numerical analysis such as

$$g(\alpha x + \beta y)(y - x)^k = a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y),$$

$$F(y)-F(x) = (y-x)[a_1 f(\alpha_1 x+\beta_1 y)+\ldots+a_n f(\alpha_n x+\beta_n y)]+(y-x)^2[g(y)-g(x)]$$

or

$$\begin{aligned}&F(y) - F(x)\\ &\quad= (y - x)\,[a_1 f(x) + b_1 f(\alpha_1 x + \beta_1 y) + \ldots + b_n f(\alpha_n x + \beta_n y) + a_1 f(y)]\\ &\qquad+ (y - x)^3[c_1 g(\alpha_1 x + \beta_1 y) + \ldots + c_n g(\alpha_n x + \beta_n y)],\end{aligned}$$

were considered in [3]. A natural question arises whether the sums

$$a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y),$$

$$a_1 f(x) + b_1 f(\alpha_1 x + \beta_1 y) + \ldots + b_n f(\alpha_n x + \beta_n y) + a_1 f(y)$$

and

$$c_1 g(\alpha_1 x + \beta_1 y) + \ldots + c_n g(\alpha_n x + \beta_n y)$$

occurring in these, more general, equations also must be continuous.

[1] B. Koclęga-Kulpa, T. Szostok, *On a class of equations stemming from various quadrature rules* Acta Math. Hungarica 2011,**130**, 340–348.

[2] A. Lisak, M. Sablik, *Trapezoidal rule revisited*, Bull. Inst. Math. Acad. Sin. (N.S.) **6** (2011), 347-–360.

[3] T. Szostok, *Functional equations stemming from numerical analysis*, submitted.

Tomasz Szostok

10. Remark.

If we assume that formulas used in the numerical differentation give exact results then the following functional equation appears

$$g(\alpha x + \beta y)(y - x)^k = a_1 f(\alpha_1 x + \beta_1 y) + \ldots + a_n f(\alpha_n x + \beta_n y). \tag{1}$$

Using a result from [1], it is possible to prove (under some assumptions) that functions $f, g\colon \mathbb{R} \to \mathbb{R}$ satisfying (1) must be polynomial functions. Since continuous polynomial functions are ordinary polynomials, we concentrate our attention on the continuity of solutions of (1).

Moreover it can be shown that if polynomial functions g, f satisfy (1) then their monomial summands of orders $p, p+k$, respectively, are also solutions of this equation. Therefore it is possible to deal with monomial functions only.

In [2] the following result has been proved.

Theorem

Let functions $f, g\colon \mathbb{R} \to \mathbb{R}$ satisfy equation (1) *Let p be a positive integer, g be a monomial function of order p, f be a monomial function of order $p+k$ and let numbers $\alpha, \beta,\ a_i, \alpha_, \beta_i, i = 1, \ldots, n$ satisfy:*

$$a_1 \alpha_1^{p+k} + \ldots + a_n \alpha_n^{p+k} \neq 0, \qquad a_1 \beta_1^{p+k} + \ldots + a_n \beta_n^{p+k} \neq 0. \tag{2}$$

Let $\mathfrak{a}_p$ be defined by

$$\mathfrak{a}_p := \frac{(-1)^k \alpha^p}{a_1 \alpha_1^{p+k} + \ldots + a_n \alpha_n^{p+k}}.$$

If the following conditions are satisfied

$$\begin{array}{lcr}
\mathfrak{a}_p \left[a_1 \alpha_1 \beta_1^{p+k-1} + \ldots + a_n \alpha_n \beta_n^{p+k-1} \right] & \neq & \alpha \beta^{p-1}, \\
\mathfrak{a}_p \left[a_1 \alpha_1^2 \beta_1^{p+k-2} + \ldots + a_n \alpha_n^2 \beta_n^{p+k-2} \right] & \neq & \alpha^2 \beta^{p-2}, \\
 & \vdots & \\
\mathfrak{a}_p \left[a_1 \alpha_1^{p-1} \beta_1^{k+1} + \ldots + a_1 \alpha_n^{p-1} \beta_n^{k+1} \right] & \neq & \alpha^{p-1} \beta, \\
\mathfrak{a}_p \left[a_1 \alpha_1^p \beta_1^k + \ldots + a_n \alpha_n^p \beta_n^k \right] & \neq & \alpha^p,
\end{array} \tag{3}$$

then functions f and g are continuous.

It is easy to see that assumptions (2) are essential, therefore we pose the following problem. Are the assumptions (3) also essential in order to obtain the continuity of f and g.

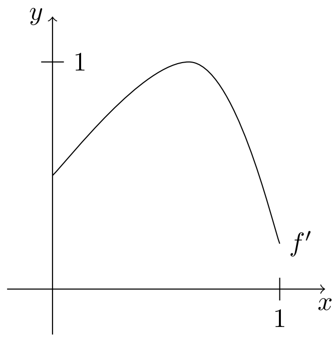
[1] A. Lisak, M. Sablik, *Trapezoidal rule revisited*, Bull. Inst. Math. Acad. Sin. (N.S.) **6** (2011), 347-–360.

[2] T. Szostok, *Functional equations stemming from numerical analysis*, submitted

Tomasz Szostok

11. Problem.

A general question was asked whether computer calculations, graphs etc. can be accepted as valid proofs of some mathematical facts.
In particular, will we accept the statement "f increases in $[0, 1]$" if the graph of its derivative looks like this:



Alfred Witkowski

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